Physics 1114 — Chapter 0: Mathematics Review

0.1 Exponents

exponent: in N^a, a is the exponent
base number: in N^a, N is the base number
squared: when the exponent is 2
cube: when the exponent is 3
square root: when the exponent is 1/2
cube root: when the exponent is 1/3
Rules for exponents are given on page 0-2.

0.2 Scientific Notation and Powers of 10

scientific notation: a form in which a quantity is expressed as a decimal number with one digit to the left of the decimal point multiplied by the appropriate power of 10.

0.3 Algebra

equation: consists of an equal sign and quantities to its left and to its right. An equation remains true if any valid operation performed on one side of the equation is also performed on the other side.

Quadratic Formula:

For a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, c are real number and $a \neq 0$, the solutions are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

simultaneous equations: if a problem has N unknowns (say x and y for 2 unknowns), it takes N equations (2 equations for 2 unknowns) to *simultaneously* solve the system. Such systems can also be solved graphically.

0.4 Direct, Inverse, and Inverse-Square Relationships

directly proportional: for two quantities, if an increase or decrease of one of the quantity causes and increase or decrease in the other by the same factor.
inverse proportion: when one quantity increases and the other quantity decreases in such a way that their product stays the same.

0.5 Logarithmic and Exponential Functions

common logarithm: the base-10 logarithm of a number y is the power to which 10 must be raised to obtain y: $y = 10^{\log y}$.
antilog: $y = 10^{\log y} = 10^x$ where x is the antilog.
natural logarithm (ln): where $e = 2.718...$ of a number y is the power to which e must be raised to obtain y: $y = e^{\ln y}$.
exponential function: $y = e^x$

0.6 Areas and Volumes

- rectangle with length a and width b has area $A = ab$
- rectangular solid with length a, width b, and height c has volume $V = abc$
- circle with radius r has diameter $d = 2r$, circumference $C = 2\pi r = \pi d$ and area $A = \pi r^2 = \pi d^2/4$
- sphere with radius r has surface area $A = 4\pi r^2$ and volume $V = \frac{4}{3}\pi r^3$
- cylinder with radius r and height h has volume $V = \pi r^2 h$

0.7 Plane Geometry and Trigonometry

similar triangle: triangles that have the same shape, but different sizes or orientations; angles are the same.
congruent: triangles that have the same size.
right triangle: one angle of the triangle is 90°
hypotenuse: the side opposite the right angle in a right triangle.
Pythagorean Theorem: $a^2 + b^2 = c^2$ where a, b are the two shortest sides of the right triangle and c is the hypothenuse.
sine:

$$\sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}}$$

cosine:

$$\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

tangent:

$$\tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$$

trigonometric functions: sine, cosine, tangent

Useful Trigonometric Identities:

$$\sin(-\theta) = -\sin(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) == \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$$

$$\sin(\theta \pm \phi) = \sin(\theta)\cos(\phi) \mp \cos(\theta)\sin(\phi)$$

$$\cos(\theta \pm \phi) = \cos(\theta)\cos(\phi) \mp \sin(\theta)\sin(\phi)$$

$$\sin(180^\circ - \theta) = \sin(\theta)$$

$$\cos(180^\circ - \theta) = -\cos(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta)$$

$$\cos(90^\circ - \theta) = \sin(\theta)$$

and for small angle θ

$$\cos(\theta) \approx 1 - \frac{\theta^2}{2} \approx 1$$

$$\sin(\theta) \approx \theta$$

Physics 1114 — Chapter 1: Models, Measurements,and Vectors

1.1 Introduction

physical theory + physical evidence → *physical laws*: principles solidly established by experimental evidence.

1.2 Idealized Models

model: in physics, a simplified version of a physical system that would be too complicated to analyze in full without the simplifications.

the validity of the predictions of the model is limited by the validity of the model

1.3 Standards and Units

physical quantity: any number that is used to describe an observation of a physical phenomenon quantitatively.
operational definition: a definition for physical quantities so fundamental that they can be defined only by describing a procedure for measuring them.
unit: the standard measurement
meter: unit measurement of length
second: unit measurement of time
kilogram: unit measure of mass
International System (SI): an internationally agreed system of units based on the metric system

Fundamental Quantities:

- Time (second)
- Length (meter)
- Mass (kilogram)
- Temperature (kelvin)
- Electric Current (ampere)
- Luminous Intensity (candela)
- Amount of Substance (mole)

1.4 Unit Consistency and Conversions

dimensionally consistent: the units of dimension must be equivalent on both sides of the equation.
conversion factor: a factor to convert one measurement into another.

Note:

Unless a quantity is specified as dimensionless, it *must* have a unit of measurement-otherwise it means *nothing.*

1.5 Precision and Significant Figures

significant figures or digits: the number of meaningful digits in a number. When numbers are multiplied or divided, the number of significant figures in the result is no greater than in the factor with the fewest significant figures. When you add or subtract, the answer can have no more decimal places than the term with the fewest decimal places.

1.6 Estimates and Orders of Magnitude

order-of-magnitude estimates: crude approximations usually to base 10.

1.7 Vectors and Vectors Addition

scalar quantity: a physical quantity described by a single number.
vector quantity: a physical quantity described by a *magnitude* (how much or how big) and a *direction* in space
displacement: a change in position of an object
vector sum or resultant: $\vec{C}$ where vectors $\vec{A} + \vec{B} = \vec{C}$
vector addition: the addition of vectors resulting in a different (magnitude and/or direction) vector
commutative: when the order of the operation does not matter; vector addition is commutative.

1.8 Components and Vectors

component vectors: for an N coordinate system, a vector can be decomposed into N ***components***. For instance in the $x - y$ plane, $\vec{A} = \vec{A}_x + \vec{A}_y$ where $\vec{A}_x$ and $\vec{A}_y$ are the components.

mechanics: the study of the relationships among force, matter, and motion
kinematics: the mathematical methods for describing motion
dynamics: the relation of motion to its causes

2.1 Displacement and Average Velocity

Definition of average velocity:

The x component of an object's average velocity is defined as the x component of displacement Δx, divided by the time interval Δt in which the displacement occurs. We represent this quantity by the letter v, with a subscript "av" to signify "average value" and a subscript "x: to signify "x component"

$$v_{av,x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}.$$

Unit: m/s

Values such as Δx or Δt denote the final value minus the initial value, always!

Average velocity is a *vector* quantity!

speed: a scalar quantity equal to the total distance traveled during a specified time interval, divided by the time interval.
slope: for the line joining two points, the "rise" divided by the "run".

The average velocity between two positions is the slope of a line connecting the two corresponding points on a graph of position as a function of time.

2.2 Instantaneous Velocity

limit: the limit of a function $f(x)$ at $x = a$ is the value which the function tends to as x approaches a
derivative: the limit of a rate of change denoted $f'(x)$ whose value is given by $\lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x}$.

Instantaneous velocity

The x component of instantaneous velocity is the limit of $\Delta x / \Delta t$ as Δt approaches zero:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}.$$

On a graph of a coordinate x as a function of time t, the instantaneous velocity at any point is equal to the slope of the tangent line to the curve at that point.

2.3 Average and Instantaneous Velocity

Average acceleration

The average acceleration a_{av} of an object as it moves from x_1 (at time t_1) to x_2 (at time t_2) is a vector quantity whose x component is the ratio of the change in the x component of velocity, $\Delta v_x = v_{2x} - v_{1x}$, to the time interval $\Delta t = t_2 - t_1$:

$$a_{av,x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}.$$

Unite: if we express velocity in meters per second and time in seconds, then the acceleration is in meters per second per second (m/s)/s. This unit is usually written as m/s^2 and is read "meters per second squared."

Instantaneous acceleration

the x component of instantaneous acceleration is defined as the limit of $\Delta v_x/\Delta t$ as Δt approaches zero:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$$

The average acceleration between any two points on a graph of velocity versus time equals the slope of a line connecting those points.
The instantaneous acceleration at any point on the graph equals the slope of the line tangent to the curve at that point.

2.4 Motion and Constant Acceleration

Velocity as a function of time for an object moving with constant acceleration

For an object that has x component of velocity v_{0x} at time $t = 0$ and moves with constant acceleration a_x, we find the velocity v_x at any later time t:

$$a_x = \frac{v_x - v_{0x}}{t - 0}, \text{or}$$

$$v_x = v_{0x} + a_x t.$$

Unit: m/s

Position as a function of time for an object moving with constant acceleration

For constant acceleration a_x, we solve:

$$v_{0x} + \frac{1}{2} a_x t = \frac{x - x_0}{t}, \quad \text{to find}$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2.$$

Unit: m/s

Velocity as a function of position for an object moving with constant acceleration

For an object moving in a straight line with constant acceleration a_x,

$${v_x}^2 = v_{0x}^2 + 2a_x(x - x_0).$$

This equation give us the particle's velocity v_x at any position x without needing to know the time when it is at that position.

Equations of motion for constant acceleration

$$v_x = v_{0x} + a_x t \quad \text{(Gives } v_x \text{ if } t \text{ is known.)}$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \quad \text{(Gives } x \text{ if } t \text{ is known.)}$$

$${v_x}^2 = v_{0x}^2 + 2a_x(x - x_0) \quad \text{(Gives } v_x \text{ if } x \text{ is known.)}$$

Position, velocity, and time for an object moving with constant acceleration

$$x - x_0 = \frac{v_{0x} + v_x}{2} t.$$

This equation says that the total displacement $x - x_0$ from time $t = 0$ to a later time t is equal to the average velocity $(v_{0x} + v_x)/2$ during the interval, multiplied by the time t.

2.5 Proportional Reasoning

$$y = kx \quad \text{(linear relation)}$$

$$y = kx^2 \quad \text{(quadratic relation)}$$

$$y = k/x \quad \text{(inverse relation)}$$

$$y = k/x^2 \quad \text{(inverse-square relation)}$$

2.6 Freely Falling Objects

free fall: idealized model in which we neglect air resistance, the earth's rotation, and the decrease in an object's acceleration with increasing altitude.
acceleration due to gravity: *or acceleration of free fall*, the constant acceleration of a freely falling object.

2.7 Relative Velocity along a Straight Line

relative velocity: the velocity seen by a particular observer is called the velocity *relative* to that observer or ***relative velocity***.
frame of reference: each observer (equipped in principle) with a meterstick and a stopwatch has their own ***frame of reference***.

3.1 Velocity in a Plane

position vector:

Average velocity:

The average velocity of a particle is the displacement $\Delta\vec{r}$, divided by the time interval Δt:

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}.$$

Instantaneous velocity $\vec{v}$ ***in a plane***:

The instantaneous velocity is the limit of the average velocity as the time interval Δt approaches zero:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}.$$

At every point along the path, the instantaneous velocity vector is tangent to the path.

3.2 Acceleration in a Plane

Average acceleration $\vec{a}_{av}$

As an object undergoes a displacement during a time interval Δt, its average acceleration is its change in velocity, $\Delta\vec{v}$, divided by Δt:

$$\text{Average acceleration} = \vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}.$$

Instantaneous acceleration $\vec{a}$

When the velocity of a particle changes by an amount $\Delta\vec{v}$ as the particle undergoes a displacement $\Delta\vec{r}$ during a time interval Δt, the instantaneous acceleration is the limit of the average acceleration as Δt approaches zero:

$$\text{Instantaneous acceleration} = \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}.$$

When a particle moves in a curved path, it always has nonzero acceleration, even when it moves with constant speed.

3.3 Projectile Motion

projectile: any object that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance.

The key to analyzing projectile motion is the fact that x and y can be treated separately.

Equations for projectile motion (assuming $a_x = 0$, $a_y = -g$)

Considering the x motion,

$$v_x = v_{0x}$$

$$x = x_0 + v_{0x}t.$$

For the y motion, we substitute y for x, v_y and v_{0y} for v_{0x}, and $-g$ for a

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

Position and velocity of a projectile as functions of time t

$$x = (v_0 \cos\theta_0)t$$

$$y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$$

$$v_x = v_0 \cos\theta_0$$

$$v_y = v_0 \sin\theta_0 - gt$$

3.4 Uniform Circular Motion

uniform circular motion: the motion of a particle in a circle with constant speed

Acceleration in uniform circular motion

The acceleration of an object in uniform circular motion is radial, meaning that it always points toward the center of the circle and is perpendicular to the object's velocity $\vec{v}$. We denote it as $\vec{a}_{rad}$; its magnitude a_{rad} is given by

$$a_{rad} = \frac{v^2}{R}.$$

That is, the magnitude a_{rad} is proportional to the square of the speed ($a_{rad} \propto v^2$) and inversely proportional to the radius ($a_{rad} \propto 1/R$).

centripetal acceleration: since the acceleration of an object in uniform circular motion is always directed toward the center of the circle, it is sometimes called centripetal acceleration.

3.5 Relative Velocity in a Plane

Relative motion in a plane

When an object W is moving with velocity $\vec{v}_{W/T}$ relative to an object (or observer) T, and T is moving with velocity $\vec{v}_{T/C}$ with respect to an object (or observer) C, the velocity $\vec{v}_{W/C}$ of W with respect to C is given by

$$\vec{v}_{W/C} = \vec{v}_{W/T} + \vec{v}_{T/C}.$$

dynamics: the relationship of motion to the forces associated with the motion.
Newton's laws of motion: three statements containing all the principles of dynamics.

4.1 Force

force: a quantitative description of the interaction between two objects or between an object and its environment.
contact force: a force involving direct contact between objects.
normal force ($\vec{n}$): a resting object possesses a component of force perpendicular to the surface upon which it rests.
friction force ($\vec{f}$): a component of force parallel to the surface that resists motion (points in the direction opposite to motion).
tension ($\vec{T}$): the force associated with a rope or cord attached to an object and pulled (you cannot push on a string!).
weight ($\vec{w}$): the gravitational attraction that the earth (or other astronomical body) exerts on an object.
resultant: net force or sum of all forces.
superposition of forces: the effect of any number of forces applied at a point on an object is the same as the effect of a single force equal to the vector sum of the original forces.

Any force can be replaced by its components, acting at the same point.

4.2 Newton's First Law

Newton's first law

Every object continues either at rest or in constant motion in a straight line, unless it is forced to change that state by forces action on it.

An object acted on by no net force moves with constant velocity (which may be zero) and thus with zero acceleration.

Zero resultant force is equivalent to no force at all.

equilibrium: an object whose vector sum (resultant) of forces is zero.
inertial frame of reference: a frame of reference with no acceleration (or acceleration to small to be of consequence).

4.3 Mass and Newton's Second Law

mass: for a given object, the ratio of the magnitude F of the force to the magnitude a of the acceleration is constant; $m = F/a$
kilogram: the SI unit of mass.

newton

One newton is the amount of force that gives an acceleration of 1 meter per second squared to an object with a mass of 1 kilogram:

$$1\text{N} = (1\text{kg})(1\text{m/s}^2).$$

Newton's second law (vector form)

The vector sum (resultant) of all the forces acting on an object equals the object's mass times its acceleration (the rate of change of its velocity):

$$\sum \vec{F} = m\vec{a}.$$

The acceleration $\vec{a}$ has the same direction as the resultant force $\sum \vec{F}$.

Newton's second law (component form)

For an object moving in a plane, each component of the total force equals the mass times the corresponding component of acceleration:

$$\sum F_x = ma_x, \quad \sum F_y = ma_y.$$

4.4 Mass and Weight

Relation of mass to weight

The weight of an object with mass m must have a magnitude w equal to the mass times the magnitude of acceleration due to gravity, g:

$$w = mg.$$

Because weight is a force, we can write this equation as a vector relation:

$$\vec{w} = m\vec{g}.$$

4.5 Newton's third Law

Newton's third law

For two interacting objects A and B, the formal statement of Newton's third law is

$$\vec{F}_{AonB} = -\vec{F}_{BonA}.$$

Newton's own statement, translated from the Latin of the *Principia*, is

> To every action there is always opposed an equal reaction; or, the mutual actions of two objects upon each other are always equal, and directed to contrary parts.

Two forces in an action-reaction pair *never* act on the same object.

4.6 Free-Body Diagrams

free-body diagram: a diagram showing the chosen object by itself, "free" of its surroundings, with vectors drawn to show the forces applied to it by the various other objects that interact with it.

Greek

alpha	α	A
beta	β	B
gamma	γ	Γ
delta	δ	Δ
epsilon	ϵ or ε	E
zeta	ζ	Z
eta	η	H
theta	θ or ϑ	Θ
iota	ι	I
kappa	κ	K
lambda	λ	Λ
mu	μ	M
nu	ν	N
xi	ξ	Ξ
omicron	o	O
pi	π or ϖ	Π
rho	ρ or ϱ	P
sigma	σ or ς	Γ
tau	τ	T
upsilon	υ	Υ
phi	ϕ or φ	Φ
chi	χ	X
psi	ψ	Ψ
omega	ω	Ω