Physics 1114 — Chapter 0: Mathematics Review

0.1 Exponents

exponent: in N^a, a is the exponent
base number: in N^a, N is the base number
squared: when the exponent is 2
cube: when the exponent is 3
square root: when the exponent is 1/2
cube root: when the exponent is 1/3
Rules for exponents are given on page 0-2.

0.2 Scientific Notation and Powers of 10

scientific notation: a form in which a quantity is expressed as a decimal number with one digit to the left of the decimal point multiplied by the appropriate power of 10.

0.3 Algebra

equation: consists of an equal sign and quantities to its left and to its right. An equation remains true if any valid operation performed on one side of the equation is also performed on the other side.

Quadratic Formula:

For a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, c are real number and $a \neq 0$, the solutions are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

simultaneous equations: if a problem has N unknowns (say x and y for 2 unknowns), it takes N equations (2 equations for 2 unknowns) to *simultaneously* solve the system. Such systems can also be solved graphically.

0.4 Direct, Inverse, and Inverse-Square Relationships

directly proportional: for two quantities, if an increase or decrease of one of the quantity causes and increase or decrease in the other by the same factor.
inverse proportion: when one quantity increases and the other quantity decreases in such a way that their product stays the same.

0.5 Logarithmic and Exponential Functions

common logarithm: the base-10 logarithm of a number y is the power to which 10 must be raised to obtain y: $y = 10^{\log y}$.
antilog: $y = 10^{\log y} = 10^x$ where x is the antilog.
natural logarithm (ln): where $e = 2.718...$ of a number y is the power to which e must be raised to obtain y: $y = e^{\ln y}$.
exponential function: $y = e^x$

0.6 Areas and Volumes

- rectangle with length a and width b has area $A = ab$
- rectangular solid with length a, width b, and height c has volume $V = abc$
- circle with radius r has diameter $d = 2r$, circumference $C = 2\pi r = \pi d$ and area $A = \pi r^2 = \pi d^2/4$
- sphere with radius r has surface area $A = 4\pi r^2$ and volume $V = \frac{4}{3}\pi r^3$
- cylinder with radius r and height h has volume $V = \pi r^2 h$

0.7 Plane Geometry and Trigonometry

similar triangle: triangles that have the same shape, but different sizes or orientations; angles are the same.
congruent: triangles that have the same size.
right triangle: one angle of the triangle is 90°
hypotenuse: the side opposite the right angle in a right triangle.
Pythagorean Theorem: $a^2 + b^2 = c^2$ where a, b are the two shortest sides of the right triangle and c is the hypothenuse.
sine:

$$\sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}}$$

cosine:

$$\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

tangent:

$$\tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$$

trigonometric functions: sine, cosine, tangent

Useful Trigonometric Identities:

$$\sin(-\theta) = -\sin(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) == \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$$

$$\sin(\theta \pm \phi) = \sin(\theta)\cos(\phi) \mp \cos(\theta)\sin(\phi)$$

$$\cos(\theta \pm \phi) = \cos(\theta)\cos(\phi) \mp \sin(\theta)\sin(\phi)$$

$$\sin(180^\circ - \theta) = \sin(\theta)$$

$$\cos(180^\circ - \theta) = -\cos(\theta)$$

$$\sin(90^\circ - \theta) = \cos(\theta)$$

$$\cos(90^\circ - \theta) = \sin(\theta)$$

and for small angle θ

$$\cos(\theta) \approx 1 - \frac{\theta^2}{2} \approx 1$$

$$\sin(\theta) \approx \theta$$

Physics 1114 — Chapter 1: Models, Measurements,and Vectors

1.1 Introduction

physical theory + physical evidence → *physical laws*: principles solidly established by experimental evidence.

1.2 Idealized Models

model: in physics, a simplified version of a physical system that would be too complicated to analyze in full without the simplifications.

> the validity of the predictions of the model is limited by the validity of the model

1.3 Standards and Units

physical quantity: any number that is used to describe an observation of a physical phenomenon quantitatively.
operational definition: a definition for physical quantities so fundamental that they can be defined only by describing a procedure for measuring them.
unit: the standard measurement
meter: unit measurement of length
second: unit measurement of time
kilogram: unit measure of mass
International System (SI): an internationally agreed system of units based on the metric system

Fundamental Quantities:

- Time (second)
- Length (meter)
- Mass (kilogram)
- Temperature (kelvin)
- Electric Current (ampere)
- Luminous Intensity (candela)
- Amount of Substance (mole)

1.4 Unit Consistency and Conversions

dimensionally consistent: the units of dimension must be equivalent on both sides of the equation.
conversion factor: a factor to convert one measurement into another.

Note:

> Unless a quantity is specified as dimensionless, it *must* have a unit of measurement-otherwise it means *nothing*.

1.5 Precision and Significant Figures

significant figures or digits: the number of meaningful digits in a number. When numbers are multiplied or divided, the number of significant figures in the result is no greater than in the factor with the fewest significant figures. When you add or subtract, the answer can have no more decimal places than the term with the fewest decimal places.

1.6 Estimates and Orders of Magnitude

order-of-magnitude estimates: crude approximations usually to base 10.

1.7 Vectors and Vectors Addition

scalar quantity: a physical quantity described by a single number.
vector quantity: a physical quantity described by a *magnitude* (how much or how big) and a *direction* in space
displacement: a change in position of an object
vector sum or resultant: $\vec{C}$ where vectors $\vec{A}+\vec{B}=\vec{C}$
vector addition: the addition of vectors resulting in a different (magnitude and/or direction) vector
commutative: when the order of the operation does not matter; vector addition is commutative.

1.8 Components and Vectors

component vectors: for an N coordinate system, a vector can be decomposed into N ***components***. For instance in the $x-y$ plane, $\vec{A}=\vec{A}_x+\vec{A}_y$ where $\vec{A}_x$ and $\vec{A}_y$ are the components.

Physics 1114 — Chapter 2: Motion along a Straight Line

mechanics: the study of the relationships among force, matter, and motion
kinematics: the mathematical methods for describing motion
dynamics: the relation of motion to its causes

2.1 Displacement and Average Velocity

Definition of average velocity:

The x component of an object's average velocity is defined as the x component of displacement Δx, divided by the time interval Δt in which the displacement occurs. We represent this quantity by the letter v, with a subscript "av" to signify "average value" and a subscript "x: to signify "x component"

$$v_{av,x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}.$$

Unit: m/s

Values such as Δx or Δt denote the final value minus the initial value, always!

Average velocity is a *vector* quantity!

speed: a scalar quantity equal to the total distance traveled during a specified time interval, divided by the time interval.
slope: for the line joining two points, the "rise" divided by the "run".

The average velocity between two positions is the slope of a line connecting the two corresponding points on a graph of position as a function of time.

2.2 Instantaneous Velocity

limit: the limit of a function $f(x)$ at $x = a$ is the value which the function tends to as x approaches a
derivative: the limit of a rate of change denoted $f'(x)$ whose value is given by $\lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x}$.

Instantaneous velocity

The x component of instantaneous velocity is the limit of $\Delta x / \Delta t$ as Δt approaches zero:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}.$$

On a graph of a coordinate x as a function of time t, the instantaneous velocity at any point is equal to the slope of the tangent line to the curve at that point.

2.3 Average and Instantaneous Velocity

Average acceleration

The average acceleration a_{av} of an object as it moves from x_1 (at time t_1) to x_2 (at time t_2) is a vector quantity whose x component is the ratio of the change in the x component of velocity, $\Delta v_x = v_{2x} - v_{1x}$, to the time interval $\Delta t = t_2 - t_1$:

$$a_{av,x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}.$$

Unite: if we express velocity in meters per second and time in seconds, then the acceleration is in meters per second per second (m/s)/s. This unit is usually written as m/s^2 and is read "meters per second squared."

Instantaneous acceleration

the x component of instantaneous acceleration is defined as the limit of $\Delta v_x/\Delta t$ as Δt approaches zero:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$$

The average acceleration between any two points on a graph of velocity versus time equals the slope of a line connecting those points.
The instantaneous acceleration at any point on the graph equals the slope of the line tangent to the curve at that point.

2.4 Motion and Constant Acceleration

Velocity as a function of time for an object moving with constant acceleration

For an object that has x component of velocity v_{0x} at time $t = 0$ and moves with constant acceleration a_x, we find the velocity v_x at any later time t:

$$a_x = \frac{v_x - v_{0x}}{t - 0}, \text{or}$$

$$v_x = v_{0x} + a_x t.$$

Unit: m/s

Position as a function of time for an object moving with constant acceleration

For constant acceleration a_x, we solve:

$$v_{0x} + \frac{1}{2} a_x t = \frac{x - x_0}{t}, \quad \text{to find}$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2.$$

Unit: m/s

Velocity as a function of position for an object moving with constant acceleration

For an object moving in a straight line with constant acceleration a_x,

$${v_x}^2 = v_{0x}^2 + 2a_x(x - x_0).$$

This equation give us the particle's velocity v_x at any position x without needing to know the time when it is at that position.

Equations of motion for constant acceleration

$$v_x = v_{0x} + a_x t \quad \text{(Gives } v_x \text{ if } t \text{ is known.)}$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \quad \text{(Gives } x \text{ if } t \text{ is known.)}$$

$$v_x{}^2 = v_{0x}^2 + 2a_x(x - x_0) \quad \text{(Gives } v_x \text{ if } x \text{ is known.)}$$

Position, velocity, and time for an object moving with constant acceleration

$$x - x_0 = \frac{v_{0x} + v_x}{2} t.$$

This equation says that the total displacement $x - x_0$ from time $t = 0$ to a later time t is equal to the average velocity $(v_{0x} + v_x)/2$ during the interval, multiplied by the time t.

2.5 Proportional Reasoning

$$y = kx \quad \text{(linear relation)}$$

$$y = kx^2 \quad \text{(quadratic relation)}$$

$$y = k/x \quad \text{(inverse relation)}$$

$$y = k/x^2 \quad \text{(inverse-square relation)}$$

2.6 Freely Falling Objects

free fall: idealized model in which we neglect air resistance, the earth's rotation, and the decrease in an object's acceleration with increasing altitude.
acceleration due to gravity: *or acceleration of free fall*, the constant acceleration of a freely falling object.

2.7 Relative Velocity along a Straight Line

relative velocity: the velocity seen by a particular observer is called the velocity *relative* to that observer or ***relative velocity***.
frame of reference: each observer (equipped in principle) with a meterstick and a stopwatch has their own ***frame of reference***.

3.1 Velocity in a Plane

position vector:

Average velocity:

The average velocity of a particle is the displacement $\Delta\vec{r}$, divided by the time interval Δt:

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}.$$

Instantaneous velocity $\vec{v}$ ***in a plane***:

The instantaneous velocity is the limit of the average velocity as the time interval Δt approaches zero:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}.$$

At every point along the path, the instantaneous velocity vector is tangent to the path.

3.2 Acceleration in a Plane

Average acceleration $\vec{a}_{av}$

As an object undergoes a displacement during a time interval Δt, its average acceleration is its change in velocity, $\Delta\vec{v}$, divided by Δt:

$$\text{Average acceleration} = \vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}.$$

Instantaneous acceleration $\vec{a}$

When the velocity of a particle changes by an amount $\Delta\vec{v}$ as the particle undergoes a displacement $\Delta\vec{r}$ during a time interval Δt, the instantaneous acceleration is the limit of the average acceleration as Δt approaches zero:

$$\text{Instantaneous acceleration} = \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}.$$

When a particle moves in a curved path, it always has nonzero acceleration, even when it moves with constant speed.

3.3 Projectile Motion

projectile: any object that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance.

The key to analyzing projectile motion is the fact that x and y can be treated separately.

Equations for projectile motion (assuming $a_x = 0$, $a_y = -g$)

Considering the x motion,

$$v_x = v_{0x}$$

$$x = x_0 + v_{0x}t.$$

For the y motion, we substitute y for x, v_y and v_{0y} for v_{0x}, and $-g$ for a

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

Position and velocity of a projectile as functions of time t

$$x = (v_0 \cos\theta_0)t$$

$$y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$$

$$v_x = v_0 \cos\theta_0$$

$$v_y = v_0 \sin\theta_0 - gt$$

3.4 Uniform Circular Motion

uniform circular motion: the motion of a particle in a circle with constant speed

Acceleration in uniform circular motion

The acceleration of an object in uniform circular motion is radial, meaning that it always points toward the center of the circle and is perpendicular to the object's velocity $\vec{v}$. We denote it as $\vec{a}_{rad}$; its magnitude a_{rad} is given by

$$a_{rad} = \frac{v^2}{R}.$$

That is, the magnitude a_{rad} is proportional to the square of the speed ($a_{rad} \propto v^2$) and inversely proportional to the radius ($a_{rad} \propto 1/R$).

centripetal acceleration: since the acceleration of an object in uniform circular motion is always directed toward the center of the circle, it is sometimes called centripetal acceleration.

3.5 Relative Velocity in a Plane

Relative motion in a plane

When an object W is moving with velocity $\vec{v}_{W/T}$ relative to an object (or observer) T, and T is moving with velocity $\vec{v}_{T/C}$ with respect to an object (or observer) C, the velocity $\vec{v}_{W/C}$ of W with respect to C is given by

$$\vec{v}_{W/C} = \vec{v}_{W/T} + \vec{v}_{T/C}.$$

dynamics: the relationship of motion to the forces associated with the motion.
Newton's laws of motion: three statements containing all the principles of dynamics.

4.1 Force

force: a quantitative description of the interaction between two objects or between an object and its environment.
contact force: a force involving direct contact between objects.
normal force ($\vec{n}$): a resting object possesses a component of force perpendicular to the surface upon which it rests.
friction force ($\vec{f}$): a component of force parallel to the surface that resists motion (points in the direction opposite to motion).
tension ($\vec{T}$): the force associated with a rope or cord attached to an object and pulled (you cannot push on a string!).
weight ($\vec{w}$): the gravitational attraction that the earth (or other astronomical body) exerts on an object.
resultant: net force or sum of all forces.
superposition of forces: the effect of any number of forces applied at a point on an object is the same as the effect of a single force equal to the vector sum of the original forces.

Any force can be replaced by its components, acting at the same point.

4.2 Newton's First Law

Newton's first law

Every object continues either at rest or in constant motion in a straight line, unless it is forced to change that state by forces action on it.

An object acted on by no net force moves with constant velocity (which may be zero) and thus with zero acceleration.

Zero resultant force is equivalent to no force at all.

equilibrium: an object whose vector sum (resultant) of forces is zero.
inertial frame of reference: a frame of reference with no acceleration (or acceleration to small to be of consequence).

4.3 Mass and Newton's Second Law

mass: for a given object, the ratio of the magnitude F of the force to the magnitude a of the acceleration is constant; $m = F/a$
kilogram: the SI unit of mass.

newton

One newton is the amount of force that gives an acceleration of 1 meter per second squared to an object with a mass of 1 kilogram:

$$1\text{N} = (1\text{kg})(1\text{m/s}^2).$$

Newton's second law (vector form)

The vector sum (resultant) of all the forces acting on an object equals the object's mass times its acceleration (the rate of change of its velocity):

$$\sum \vec{F} = m\vec{a}.$$

The acceleration $\vec{a}$ has the same direction as the resultant force $\sum \vec{F}$.

Newton's second law (component form)

For an object moving in a plane, each component of the total force equals the mass times the corresponding component of acceleration:

$$\sum F_x = ma_x, \quad \sum F_y = ma_y.$$

4.4 Mass and Weight

Relation of mass to weight

The weight of an object with mass m must have a magnitude w equal to the mass times the magnitude of acceleration due to gravity, g:

$$w = mg.$$

Because weight is a force, we can write this equation as a vector relation:

$$\vec{w} = m\vec{g}.$$

4.5 Newton's third Law

Newton's third law

For two interacting objects A and B, the formal statement of Newton's third law is

$$\vec{F}_{AonB} = -\vec{F}_{BonA}.$$

Newton's own statement, translated from the Latin of the *Principia*, is

> To every action there is always opposed an equal reaction; or, the mutual actions of two objects upon each other are always equal, and directed to contrary parts.

Two forces in an action-reaction pair *never* act on the same object.

4.6 Free-Body Diagrams

free-body diagram: a diagram showing the chosen object by itself, "free" of its surroundings, with vectors drawn to show the forces applied to it by the various other objects that interact with it.

5.1 Equilibrium of a Particle

equilibrium: when an object is at rest or moving with constant velocity in an inertial frame of reference.

When an object is at rest or is moving with constant velocity in an inertial frame of reference, the vector sum of all the forces acting on it must be zero.

Necessary condition for equilibrium of an object

For an object to be in equilibrium, the vector sum of the forces acting on it must be zero:

$$\sum \vec{F} = 0.$$

This condition is sufficient only if the object can be treated as a particle, which we assume in the next principle.

Equilibrium conditions in component form

An object is in equilibrium if the sum of the components of force in each axis direction is zero:

$$\sum F_x = 0 \quad \sum F_y = 0.$$

5.2 Applications of Newton's Second Law

Newton's second law

An object's acceleration equals the vector sum of the forces acting on it, divided by its mass. In vector form, we rewrite this statement as

$$\sum \vec{F} = m\vec{a}.$$

In the component form:

$$\sum F_x = ma_x, \quad \sum F_y = ma_y.$$

5.3 Contact Forces and Friction

kinetic friction: the associated friction force when an object is sliding with respect to a surface.

Relation between kinetic-friction force and normal force

When the magnitude of the sliding friction force f_k is roughly proportional to the magnitude n of the normal force, the two are related by a contestant μ_k called the coefficient of kinetic friction:

$$f_k = \mu_k n.$$

Because u_k is the ratio of two force magnitudes, it has no units.

static-friction force: the associated friction force when an object is at rest.

Relation between normal force and maximum static-friction force

when the maximum magnitude of the static-friction force can be represented as proportional to the magnitude of the normal force, the two are related by a constant μ_s called the coefficient of static friction:

$$f_s \leq \mu_s n.$$

coefficient of rolling friction: μ_r, the horizontal force needed for constant speed on a flat surface, divided by the upward normal force exerted by the surface.

5.4 Elastic Forces

Elastic behavior of springs (Hooke's law)

For springs, the spring force F_{spr} is approximately proportional to the distance x by which the spring is stretched or compressed:

$$F_{spr} = -kx.$$

force constant: k, the positive proportionality constant; spring constant of the spring; units of N/m.

6.1 Force in Circular Motion

Acceleration in uniform circular motion

When an object moves at constant speed around a circle (or an arc of a circle) the magnitude of its acceleration (a_{rad}) is proportional to the square of its speed v and inversely proportional to the radius R of the circle:

$$a_{rad} = \frac{v^2}{R}.$$

period: the time for one revolution.

6.2 Motion in a Vertical Circle

6.3 Newton's Law of Gravitation

Newton's law of gravitation

Newton's law of gravitation may be stated as follows:

> Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.

For two particles with masses m_1 and m_2, separated by a distance r, Newton's law of gravitation can be expressed as

$$F_g = G\frac{m_1 m_2}{r^2},$$

where F_g is the magnitude of the gravitational force on either particle and G is a fundamental physical constant called the **gravitational constant**. Gravitational forces are always attractive; gravitation always pulls objects toward each other.

Gravitational Constant via NIST as of 6/9/13 is $6.67384 \cdot 10^{-11}$ m^3/ kg s^2

6.4 Weight

The weight of an object is the total gravitational force exerted on the object by all other objects in the universe.

$$w = F_g = G\frac{m m_E}{R_E^2} \quad \text{near earth's surface}$$

6.5 Satellite Motion

7.1 An Overview of Energy

conservation of energy: the total energy in any isolated system is constant, no matter what happens within the system.
kinetic energy: for a particle of mass m moving with speed v, $KE = \frac{1}{2}mv^2$.
mechanical energy: energy associated with motion, position, and deformation of objects. Kinetic, elastic potential, and gravitational potential energy are all forms of mechanical energy.
potential energy: stored energy.
elastic potential energy: energy stored in an elastic object when it is stretched, compressed, twisted, or otherwise deformed.
gravitational potential energy: stored energy due to an elevated position.
system: in energy relations, a system usually consists of one or more objects that can interact, move, and undergo deformations.
isolated system: a special case where a system has no interaction with its surroundings.

The total energy in any isolated system is constant.

internal energy: form of energy due to atomic-scale vibration.
dissipative: any force, such as friction, that turns mechanical energy into nonmechanical forms of energy.

In the book in Table 7.1, "How money is not like energy:" additionally, money could be eradicated from earth tomorrow and life would continue as normal, however if energy were eradicated, life, all matter would cease to exist, ($E = mc^2$).

7.2 Work

Work

When an object undergoes a displacement $\vec{s}$ with magnitude s along a straight line, while a constant force $\vec{F}$ with magnitude F, making an angle ϕ with $\vec{s}$, acts on the object, the work done by the force on the object is

$$W = F_{||}s = (F\cos\phi)s.$$

Unit: joule (J)

$$1 \text{ joule } = 1 \text{ newton-meter} \quad \text{ or } 1 \text{ J} = 1 \text{ N} \cdot \text{m}.$$

7.3 Work and Kinetic Energy

Work-energy theorem

The kinetic energy K of a particle with mass m moving with speed v is $K = \frac{1}{2}mv^2$. During any displacement of the particle, the work done by the net external force on it is equal to its change in kinetic energy, or

$$W_{total} = K_f - K_i = \Delta K.$$

$$1 \text{ J } = 1 \text{ N} \cdot \text{m } = 1(\text{kg} \cdot \text{m/s}^2) \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2.$$

7.4 Work Done by a Varying Force

On a graph of force as a function of position, the total work done by the force is represented by the area under the curve between the initial and final positions.

7.5 Potential Energy

conservative forces: forces that can be associated with a potential energy
conservative system: a system in which the total mechanical energy, kinetic and potential, is constant.

Gravitational potential energy

When an object with mass m is a vertical distance y above the origin of coordinates, in a uniform gravitational field g, the gravitational potential energy U_{grav} of the system is

$$U_{grav} = mgy.$$

Like work and all forms of energy, gravitational potential energy is measured in joules.

total mechanical energy: $K + U$, kinetic plus potential energy of the system.

When $\vec{F}_{other} = 0$, the total mechanical energy is constant or conserved.

spring force: the work done by the force that the spring exerts on an object.

Elastic potential energy

When a spring that obeys Hooke's law, $F = kx$, is stretched or compressed a distance x from its undistorted state, the associated potential energy U_{el} is given by

$$U_{el} = \frac{1}{2}kx^2.$$

7.6 Conservation of Energy

$$K_f - K_i = U_{grav,i} - U_{grav,f} + U_{el,i} - U_{el,f} + W_{other}$$

7.7 Conservative and Non-conservative Forces

conservative force: a force that permits two-way conversion between kinetic and potential energies; always reversible!
nonconservative or dissipative force: unrecoverable energy; energy lost, frequently as heat.

7.8 Power

power: time rate at which work is done or energy is transferred.

Average power

When a quantity of work ΔW is done during a time interval Δt, the average power P_{av}, or work per unit time, is defined as

$$P_{av} = \frac{\Delta W}{\Delta t}.$$

Unit: watt (W), where 1 watt is defined as 1 joule per second (J/s).

instantaneous power: the limit of P as Δt goes to zero, $P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$.

8.1 Momentum $\vec{p}$

Momentum

The momentum of a particle, denoted by $\vec{p}$ is the product of its mass m and its velocity $\vec{v}$:

$$\vec{p} = m\vec{v}.$$

Unit: kg · m/s.

Newton's second law in terms of momentum

The vector sum of forces acting on a particle equals the rate of change of momentum of the particle with respect to time:

$$\sum \vec{F} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}.$$

Total Momentum

The total momentum of two or more particles is the vector sum of the momenta (plural of momentum) of the particles, thus, if two particles A and B have momenta $\vec{p}_A$ and $\vec{p}_B$, their total momentum $\vec{P}$ is the vector sum

$$\vec{P} = \vec{p}_A + \vec{p}_B.$$

This can be extended to any number of particles so that

$$\vec{P} = \vec{p}_A + \vec{p}_B + \vec{p}_C + \cdots . \quad \text{(total momentum of a system of particles)}$$

8.2 Conservation of Momentum

internal forces: for any system (collection of particles), the various particles exert forces on each other.
external forces: forces exerted on any part of the system by objects outside the system.
isolated system: a system that is acted upon by no external forces.

Internal forces cannot change the total momentum of a system.

The total momentum of an isolated system is constant.

Conservation of Momentum

The total momentum of a system is constant whenever the vector sum of the external forces on the system is zero. In particular, the total momentum of an isolated system is constant.

8.3 Inelastic Collisions

elastic collision: when the interaction forces between the objects are conservative, the total kinetic energy of the system is the *same* after the collision as before.
inelastic collision: a collision in which the total kinetic energy after the collision is *less* than that before the collision.

8.4 Elastic Collisions

In an elastic collision, the relative velocity of the two objects has the same magnitude before and after the collision, and the two relative velocities has opposite directions.

8.5 Impulse

Impulse

When a constant force $\vec{F}$ acts on an object, the impulse of the force, denoted by $\vec{J}$, is the force multiplied by the time interval during which it acts:

$$\vec{J} = \vec{F}(t_f - t_i) = \vec{F}\Delta t.$$

The impulse is a vector quantity–its direction is the same as that of the $\vec{F}$.
Unit: Force time time (N · s).

Impulse-momentum theorem: relation of impulse to change in momentum

When a constant force $\vec{F}$ acts on an object during a time interval $\Delta t = t_f - t_i$, the change in the object's momentum is equal to the impulse of the force acting on the object, or

$$\Delta\vec{p} = \vec{F}\Delta t - \vec{F}(t_f - t_i) = \vec{J}.$$

Unit: mass times velocity (kg · m/s)

8.6 Center of Mass

Center of mass

Suppose there are several particles A, B,..., with masses m_A,m_B,.... Let the coordinates of A be (x_A,y_A), let those of B be (x_B,y_B), and so on. The *center of mass* of the system is the point having coordinates (x_{cm},y_{cm}) given by

$$x_{cm} = \frac{m_A x_A + m_B x_B + m_C x_C + \cdots}{m_A + m_B + m_C + \cdots},$$

$$y_{cm} = \frac{m_A y_A + m_B y_B + m_C y_C + \cdots}{m_A + m_B + m_C + \cdots}.$$

Velocity of center of mass

The velocity $\vec{v}_{cm}$ of the center of mass of a collection of particles is the mass-weighted average of the velocities of the individual particles:

$$\vec{v}_{cm} = \frac{m_A \vec{v}_A + m_B \vec{v}_B + m_C \vec{v}_C + \cdots}{m_A + m_B + m_C + \cdots}.$$

In terms of components,

$$v_{cm,x} = \frac{m_A v_{A,x} + m_B v_{B,x} + m_C v_{C,x} + \cdots}{m_A + m_B + m_C + \cdots},$$

$$v_{cm,y} = \frac{m_A v_{A,y} + m_B v_{B,y} + m_C v_{C,y} + \cdots}{m_A + m_B + m_C + \cdots}.$$

Total momentum $\vec{P}$ in terms of center of mass

For a system of particles, the total momentum $\vec{P}$ is the total mass $M = m_A + m_B + \cdots$ times the velocity $\vec{v}_{cm}$ of the center of mass:

$$M\vec{v}_{cm} = m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C + \cdots = \vec{P}.$$

8.7 Motion of the Center of Mass

Acceleration of center of mass

When an object or a collection of particles is acted on by external forces, the center of mass moves just as though all the mass were concentrated at that point and were acted on by a resultant force equal to the sum of the external forces on the system. Symbolically:

$$\sum \vec{F}_{ext} = M\vec{a}_{cm}.$$

8.8 Rocket Propulsion

9.1 Angular Velocity and Angular Acceleration

Average angular velocity

When a rigid body rotates through an angular displacement $\Delta\theta = \theta_2 - \theta_1$ in a time interval $\Delta t = t_2 - t_1$, the average angular velocity ω_{av} is defined as the ratio of the angular displacement to the time interval:

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}.$$

Unit: rad/s

Instantaneous angular velocity

The instantaneous angular velocity of ω is the limit of ω_{av} as Δt approaches zero:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}.$$

At any given instant, every part of a rigid body has the same angular velocity.

$$1 \text{ rev/s } = 2\pi \text{ rad/s} \quad \text{and} \quad 1 \text{ rev/min } = 1 \text{ rpm } = \frac{2\pi}{60}\text{rad/s}$$

Average angular acceleration

The average angular acceleration α_{av} of a rotating body is the change in angular velocity, $\Delta\omega$, divided by the time interval Δt:

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}.$$

Unit: rad/s^2

Instantaneous angular acceleration

The instantaneous angular acceleration of α is the limit of α_{av} as Δt approaches zero:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}.$$

9.2 Rotation with Constant Angular Acceleration

Comparison of linear and angular motion with constant acceleration

Straight-line motion with constant linear acceleration	Fixed-axis rotation with constant angular acceleration
$a =$ constant	$\alpha =$ constant
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\omega - \omega_0)$
$x - x_0 = \frac{1}{2}(v + v_0)t$	$\theta - \theta_0 = \frac{1}{2}(\omega + \omega_0)t$

9.3 Relationship between Linear and Angular Quantities

The acceleration $\vec{a}$ of a particle can be represented by its radial and tangential components.

tangential component of instantaneous acceleration

$$a_{tan} = r\alpha$$

radial component of instantaneous acceleration

$$a_{rad} = \frac{v^2}{r} = \omega^2 r$$

$$|\vec{a}| = a = \sqrt{a_{tan}^2 + a_{rad}^2}$$

9.4 Kinetic Energy of Rotation and Moment of Inertia

Moment of inertia

A body's moment of inertia, I, describes how its mass is distributed in relation to an axis of rotation:

$$I = m_A r_A^2 + m_B r_B^2 + m_C r_C^2 + \cdots .$$

Unit: $\text{kg} \cdot \text{m}^2$

9.5 Rotation about a Moving Axis

1. Every possible motion of a rigid body can be represented as a combination of motion of the center of mass and rotation about an axis through the center of mass.
2. The total kinetic energy can always be represented as the sum of a part associated with motion of the center of mass, treated as a point, plus a part associated with rotation about an axis through the center of mass.

$$K_{total} = \frac{1}{2}M\vec{v}_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

10.1 Torque

Torque

Torque is a quantitative measure of the tendency of a force to cause or change rotational motion around a chosen axis. Torque is the product of the magnitude of the force and the moment arm, which is the perpendicular distance between the axis and the line of action of the force:

$$\tau = Fl.$$

Unit: newton-meter (N · m)

In a uniform gravitational field, the total torque on a body due to its weight is the same as though all the weight were concentrated at the center of mass of the body.

10.2 Torque and Angular Acceleration

Relation of torque to angular acceleration

The net torque (about a chosen axis) of all the forces acting on a rigid body equals the body's moment of inertia (about that axis), multiplied by its angular acceleration:

$$\sum \tau = I\alpha.$$

The torques due to all the internal forces add to zero.

10.3 Work and Power in Rotational Motion

Work done by a constant torque

The work done on a body by a constant torque equals the product of the torque and the angular displacement of the body:

$$W = \tau(\theta_2 - \theta_1) = \tau\Delta\theta.$$

Unit: If τ is expressed in newton-meters (N · m) and θ in radians, the work is in joules.

10.4 Angular Momentum

Angular momentum

When a rigid body with moment of inertia I (with respect to a specified symmetry axis) rotates with angular velocity ω about that axis, the angular momentum of the body with respect to the axis is the product of the moment of inertia I about the axis and the angular velocity ω. The angular momentum, L:

$$L = I\omega.$$

The sign of angular momentum depends on the sign of ω; thus, according to our usual convention, it is positive for counterclockwise rotation and negative for clockwise rotation. Unit: kg · m^2/s.

Torque and rate of change of angular acceleration

The rate of change of angular momentum of a rigid body with respect to any axis equals the sum of the torques of the forces acting on it with respect to that axis:

$$\sum \tau = \frac{\Delta L}{\Delta t}$$

Greek

alpha	α	A
beta	β	B
gamma	γ	Γ
delta	δ	Δ
epsilon	ϵ or ε	E
zeta	ζ	Z
eta	η	H
theta	θ or ϑ	Θ
iota	ι	I
kappa	κ	K
lambda	λ	Λ
mu	μ	M
nu	ν	N
xi	ξ	Ξ
omicron	o	O
pi	π or ϖ	Π
rho	ρ or ϱ	P
sigma	σ or ς	Γ
tau	τ	T
upsilon	υ	Υ
phi	ϕ or φ	Φ
chi	χ	X
psi	ψ	Ψ
omega	ω	Ω