Math 2283 - Introduction to Logic

Quiz #10 - 2012.09.19

Solutions

Remember that we have proved, in this chapter, the following theorems:

Theorem I: Leibniz's Law
Theorem II: $x = x$
Theorem III: $x = y \to y = x$
Theorem IV: $(x = y \wedge y = z) \to x = z$
Theorem V: $(x = z \wedge y = z) \to x = y$
Theorem VI: $(x = y \wedge y = z \wedge z = t) \to x = t$

Use Theorems I-VI along with R.O.D. and any other known logical laws to prove:

Theorem VII: $(x = y \wedge y = z \wedge t = z) \to t = x$

Proof:
First we assume

(1) $x = y \wedge y = z \wedge t = z$

Then by using an instance of the law of and $p \wedge q \wedge r \to p \wedge q$ with $p : x = y$, $q : y = z$ and $r : t = z$ with R.O.D. gives

(2) $x = y \wedge y = z$

Similarly, by using an instance of the law of and $p \wedge q \wedge r \to r$ with $p : x = y$, $q : y = z$ and $r : t = z$ with R.O.D. gives

(3) $t = z$

An exact use of Theorem IV with (2) and R.O.D. gives

(4) $x = z$

The Law of And(M) with (4) and (3) with R.O.D. yields

(5) $x = z \wedge t = z$

An instance of Theorem V with $x : x$, $z : z$ and $y : t$ is

(6) $x = t \wedge t = z \to x = t$

Using R.O.D. on (6) and (5) gets us super close to our goal:

(7) $x = t$

Finally, we use an instance of Theorem III, with $y : t$ and $x : x$

(8) $x = t \to t = x$

Using R.O.D. on (8) and (7) gives

(9) $t = x$

Rejoice! □