

Math 4133 - Linear Algebra

Quiz #18 - 2013.03.13

Solutions

Let \mathbf{S}_2 be the standard basis for \mathbb{R}^2 , and let $\mathbf{B} = \{\langle 1, -1 \rangle, \langle 1, 1 \rangle\}$.

1. Write the vector $\langle 2, 3 \rangle_{\mathbf{B}}$ in terms of the standard basis \mathbf{S}_2

$$\begin{aligned}\langle 2, 3 \rangle_{\mathbf{B}} &= 2\langle 1, -1 \rangle + 3\langle 1, 1 \rangle \\ &= \langle 2, -2 \rangle + \langle 3, 3 \rangle \\ &= \langle 5, 1 \rangle\end{aligned}$$

2. Write the vector $\langle 2, 3 \rangle$ in terms of the basis \mathbf{B}

We can do this in a multitude of ways, but the simplest for such a small problem is to write

$$\langle 2, 3 \rangle = a\langle 1, -1 \rangle + b\langle 1, 1 \rangle$$

which results in the two equations $a + b = 2$ and $-a + b = 3$. Solving gives $b = \frac{5}{2}$ and $a = -\frac{1}{2}$. Thus

$$\langle 2, 3 \rangle = \left\langle -\frac{1}{2}, \frac{5}{2} \right\rangle_{\mathbf{B}}$$