Math 4133 - Linear Algebra

Quiz #19 - 2013.03.25

Solutions

Consider the set of functions $\mathbf{F} = \{1, x, x^3\}$, and the set of points $S = \{(0,0), (-1,1), (1,4), (2,3), (-2,2)\}$.

1. Would you expect there to be a function which is a linear combination of functions from $\mathbf{F}$ that passes through the data set S exactly?

No, there are 3 unknown constants, and 5 data points. To be exact, there should be the same number of points as unknown constants in the linear combination of functions.

2. Set up the system of equations which would determine whether or not your answer to part 1 is correct.

So, a function would be $F(x) = a + bx + cx^3$. We want $F(0) = 0$, $F(-1) = 1$, $F(1) = 4$, $F(2) = 3$ and $F(-2) = 2$. This gives the system of equations

$$\begin{aligned} a &= 0 \\ a - b - c &= 1 \\ a + b + c &= 4 \\ a + 2b + 8c &= 3 \\ a - 2b - 8c &= 2 \end{aligned}$$

Writing this in matrix form, we have

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & 8 \\ 1 & -2 & -8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 4 \\ 3 \\ 2 \end{bmatrix}$$

3. Compute the matrix for which you would take the determinant to compute the Wronskian of the set $\mathbf{F}$.

$$W(1, x, x^3) = \det\left(\begin{bmatrix} 1 & x & x^3 \\ 0 & 1 & 3x^2 \\ 0 & 0 & 6x \end{bmatrix} \right)$$