Physics 1214 - General Physics II

Midterm - 2013.03.13

Solutions

Instructions: Please show all work on each problem, and give full explanations where needed. No points will be awarded for a correct answer, points are awarded on the work shown for each problem. When you are finished, please attach your cheat sheet to this exam. Good luck!

problem	points	score
1 (a)	8	
1 (b)	8	
2 (a)	5	
2 (b)	5	
3 (a)	5	
3 (b)	5	
3 (c)	5	
3 (d)	5	
4 (a)	7	
4 (b)	3	
5 (a)	7	
5 (b)	5	
6 (a)	7	
6 (b)	3	
7 (a)	5	
7 (b)	5	
Total	88	

1. Within the nucleus of an atom, compute the strength of the electric field produced, and the force exerted by a proton (a) at a distance of 5.0×10^{-15} m away from another proton, and (b) at a distance of 5.0×10^{-10} m away from an electron.

(a) We first compute the strength of the electric field for the proton-proton interaction, using the formula $E = k\frac{|q|}{r^2}$:

$$E_{p-p} = (8.99 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2)\frac{1.6 \times 10^{-19}\mathrm{C}}{(5.0 \times 10^{-15}\mathrm{m})^2} = 5.8 \times 10^{19}\mathrm{N/C}$$

We next compute the strength of the force exerted by the proton and another proton using $F = k\frac{|q_1 q_2|}{r^2}$:

$$F_{p-p} = (8.99 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2)\frac{(1.6 \times 10^{-19}\mathrm{C})^2}{(5.0 \times 10^{-15}\mathrm{m})^2} = 9.2\mathrm{N}$$

(b) Secondly, we compute the strength of the electric field for the proton-electron interaction, using the formula $E = k\frac{|q|}{r^2}$:

$$E_{p-e} = (8.99 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2)\frac{1.6 \times 10^{-19}\mathrm{C}}{(5.0 \times 10^{-10}\mathrm{m})^2} = 5.8 \times 10^{9}\mathrm{N/C}$$

We next compute the strength of the force exerted by the proton and the electron using $F = k\frac{|q_1 q_2|}{r^2}$:

$$F_{p-e} = (8.99 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2)\frac{(1.6 \times 10^{-19}\mathrm{C})^2}{(5.0 \times 10^{-10}\mathrm{m})^2} = 9.2 \times 10^{-10}\mathrm{N}$$

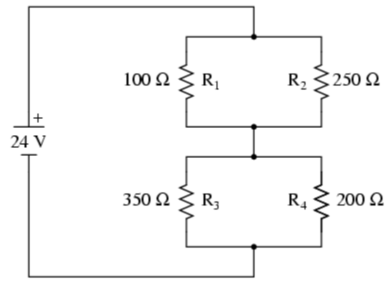
2. A point charge has a charge of 2.50×10^{-11} C. At what distance from the point charge is the electric potential (a) 90.0 V and (b) 30.0 V?

We use the formula $V = k\frac{q}{r}$ for both parts, solving for r first:

$$(a)\ r_{90} = (8.99{\times}10^9 \mathrm{N{\cdot}m^2/C^2})\frac{2.50 \times 10^{-11}}{90.0\mathrm{V}} = 2.50{\times}10^{-3}\mathrm{m}$$

$$(b)\ r_{30} = (8.99{\times}10^9 \mathrm{N{\cdot}m^2/C^2})\frac{2.50 \times 10^{-11}}{30.0\mathrm{V}} = 7.50{\times}10^{-3}\mathrm{m}$$

3. Refer to the circuit diagram given to answer the following questions.



Circuit diagram for problem 3.

(a) Find the equivalent resistance R_{12} to the combination of resistors R_1 and R_2.

Since R_1 and R_2 are in parallel,

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_{12} = \frac{500}{7}\Omega \approx 71.5\Omega$$

(b) Find the equivalent resistance R_{34} to the combination of resistors R_3 and R_4.

Since R_3 and R_4 are in parallel,

$$\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} \rightarrow R_{34} = \frac{1400}{11}\Omega \approx 127.3\Omega$$

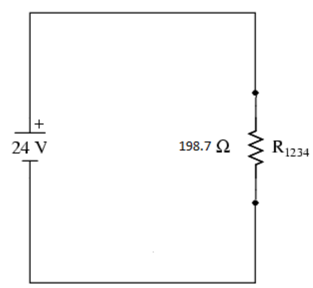
(c) Find the equivalent resistance R_{1234} to the combination of resistors R_1, R_2, R_3 and R_4.

We use R_{12} and R_{34} in series, thus

$$R_{1234} = R_{12} + R_{34} \rightarrow R_{1234} = \frac{15300}{7}\Omega \approx 198.7\Omega$$

(d) Find the current through the 24 V battery.

It is much easier to consider the following diagram, based on part (c), to answer part (d):



Circuit diagram for part (d).

Since the current through the battery is the same as the current through the network, and R_{1234} is equivalent to the combination of R_1, R_2, R_3 and R_4, we have that $V = IR$ where $V = 24$ Volts, and $R = 198.7\Omega$, thus

$$I = \frac{V}{R} = \frac{24\text{ V}}{198.7\Omega} = 0.12\text{ A}$$

4. A long straight cable contains 30 wires, each carrying a current of 1.25 A. The distances between the wires is negligible. (a) If the currents in all 30 wires are in the same direction, what is the magnitude of the magnetic field 6.0 m from the cable? (b) If the currents in half the cables is reversed, what is the new magnitude of the magnetic field 6.0 m from the cable?

For both parts, we use the formula for magnitude of a magnetic field for a long straight wire, given by $B = \frac{\mu_0 I}{2\pi r}$. For (a), the currents are all in the same direction, thus

$$B = 30\frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(1.25\ \mathrm{A})}{2\pi(6.0\ \mathrm{m})} = 1.25 \cdot 10^{-6}\ \mathrm{T}$$

For (b), since the number of wires with current in opposite directions are equal, the strength of the magnetic field is zero.

5. One solenoid is centered inside another. The outer solenoid has a length of 50.0 cm and contains 6750 coils, while the coaxial inner solenoid is 3.0 cm long and 0.120 cm in diameter and contains 15 coils. The current in the outer solenoid is changing at 37.5 A/s. (a) What is the mutual inductance of these solenoids? (b) Find the emf induced in the inner solenoid.

(a) The mutual inductance of two solenoids is given by $M = \dfrac{\mu_0 A N_1 N_2}{l}$, where A and l are the cross-sectional area inside the solenoids and length of the outer solenoid, respectively.

$$M = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})\pi(6.0 \times 10^{-4}\ \mathrm{m})^2(6750)(15)}{0.500\ \mathrm{m}} = 2.99 \times 10^{-7}\ \mathrm{H}$$

(b) To compute $\mathcal{E}_{\text{inner}}$, we use $\mathcal{E}_{\text{inner}} = M \left| \dfrac{\Delta i_{\text{outer}}}{\Delta t} \right|$:

$$\mathcal{E}_{\text{inner}} = M \left| \frac{\Delta i_{\text{outer}}}{\Delta t} \right| = 2.99 \times 10^{-7}\ \mathrm{H} \cdot 37.5\ \mathrm{A/s} = 1.08 \times 10^{-5}\mathrm{V}$$

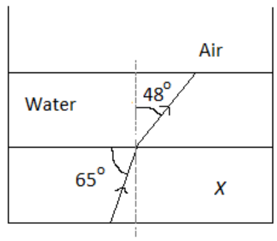
6. A series ac circuit has a resonance angular frequency of 1525 rad/s using a 138 Ω resistor, a 10.5 μF capacitor, and an inductor. (a) What is the inductance of the inductor? (b) What is the impedance of this circuit when you in use with an ac voltage source having angular frequency 1525 rad/s?

(a) For the inductance, $L = \dfrac{1}{C\omega_0^2}$:

$$L = \frac{1}{(10.5 \times 10^{-6}\ \mathrm{F})(1525\ \mathrm{rad/s})} = 0.041\ \mathrm{H}$$

(b) The impedance of the circuit satisfies $Z = R$ at resonance, thus $Z = R = 138\Omega$.

7. A layer of water covers a slab of material X in a container. A ray of light traveling upwards follows the path in the image below. The angles in the figure are given by 65° in the unknown material X and 48° in the water. (a) Compute the index of refraction of the unknown material X and (b) the angle the light makes with the normal in the air. Remember that $n_{air} = 1.0$ and $n_{water} = 1.333$.



Cross section of beaker for problem 7.

(a) We apply Snell's Law, with $n_X = n_w \dfrac{\sin(\theta_w)}{\sin(\theta_X)}$. Note that $\theta_w = 48°$ directly from the picture, but $\theta_X = 90° - 65° = 25°$. Thus we have

$$n_X = n_w \frac{\sin(\theta_w)}{\sin(\theta_X)} = 1.333 \cdot \frac{\sin(48°)}{\sin(25°)} = 2.34$$

(b) We apply Snell's Law again, with $\sin(\theta_{air}) = \dfrac{n_w}{n_{air}} \sin(\theta_w)$:

$$\sin(\theta_{air}) = \frac{n_w}{n_{air}} \sin(\theta_w) = \frac{1.333}{1.00} \sin(48°) = 0.991$$

Thus $\theta_{air} = \sin^{-1}(0.991) \approx 82°$.