

## 1 Induction Experiments

**emf or electromotive force:** (from Chapter 19) the influence that moves charge from lower to higher potential.

**induced current:** changing magnetic field causing a current in a circuit.

**induced emf:** the emf that is present to cause a current.

## 2 Magnetic Flux

### *Magnetic flux through a plane surface in a constant magnetic field*

For a plane surface with area  $A$  in a uniform magnetic field  $\vec{B}$ , both  $B_{\perp}$  and  $\phi$  are the same at all points on the surface. The magnetic flux  $\Phi_B$  through the surface is

$$\Phi_B = B_{\perp} A = BA \cos \phi.$$

If  $\vec{B}$  happens to be perpendicular to the surface, then  $\cos \phi = 1$  and the previous equation reduces to

$$\Phi_B = BA.$$

Unit: The SI unit magnetic flux is the unit magnetic field (1 T) times the unit of area (1 m<sup>2</sup>):

$$(1 \text{ T})(1 \text{ m}^2) = [1 \text{ N}/(\text{A} \cdot \text{m})](1 \text{ m}^2) = 1 \text{ N} \cdot \text{m}/\text{A}.$$

This unit is call 1 weber (1 Wb).

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2 = 1 \text{ N} \cdot \text{m}/\text{A}.$$

### *magnetic flux density*

$$B = \frac{\Delta \Phi_B}{\Delta A}$$

$$1 \text{ T} = 1 \text{ Wb}/\text{m}^2$$

## 3 Faraday's Law

The common element in ALL induction effects is changing magnetic flux through a circuit.

### *Faraday's law of induction*

The magnitude of the induced emf in a circuit equals the absolute value of the time rate of change of the magnetic flux through the circuit.

In symbols, Faraday's law is

$$\mathcal{E} = \left| \frac{\Delta \Phi_B}{\Delta t} \right|.$$

In this definition,  $\mathcal{E}$  is the magnitude of the emf and is always positive.

For a coil with  $N$  identical turns:

$$\mathcal{E} = N \left| \frac{\Delta \Phi_B}{\Delta t} \right|.$$

**generator:** a device that converts mechanical energy to electrical energy.

## 4 Lenz's Law

### *Lenz's Law*

The direction of any magnetically induced current or emf is such as to oppose the direction of the phenomenon causing it.

Induced current tends to preserve the *status quo* by opposing the motion or the change of flux that originally induced it.

## 5 Motional Electromotive Force

**motional emf**: emf created by motion of a conductor; denoted by  $\mathcal{E}$ . When velocity ( $v$ ), magnetic field ( $B$ ), and length ( $L$ ) are mutually perpendicular ( $\perp$ )

$$\mathcal{E} = vBL.$$

## 6 Eddy Currents

**eddy currents**: induced currents circulating throughout the volume of a conducting material; resulting flow patterns resembles swirling eddies in a fluid.

## 7 Mutual Inductance

### *Mutual inductance*

The mutual inductance  $M$  of two coils is given by

$$M = M_{21} = M_{12} = \left| \frac{N_2 \Phi_{B2}}{i_1} \right| = \left| \frac{N_1 \Phi_{B1}}{i_2} \right|.$$

From the preceding analysis, we can also write

$$\mathcal{E}_2 = M \left| \frac{\Delta i_1}{\Delta t} \right| \quad \text{and} \quad \mathcal{E}_1 = M \left| \frac{\Delta i_2}{\Delta t} \right|.$$

Unit: The SI unit of mutual inductance is called the **henry** (1 H).

$$1 \text{ H} = 1 \text{ Wb/A} = 1 \text{ V} \cdot \text{s/A} = 1 \Omega \cdot \text{s}.$$

## 8 Self Inductance

*self-induced emf*: an induced emf in a circuit resulting from the variation of its own magnetic field.

### *Inductance or self-inductance*

The self-inductance  $L$  of a circuit is the magnitude of the self-induced emf  $\mathcal{E}$  per unit rate of change of current, so that:

$$\mathcal{E} = L \left| \frac{\Delta i}{\Delta t} \right|$$

where the inductance  $L$  is given by

$$L = \left| \frac{N \Phi_B}{i} \right|$$

and where there are  $N$  turns of the wire carrying a current  $i$ , and  $\Phi_B$  is the magnetic flux. The SI unit of self-inductance is the same as for mutual inductance, one **henry**.

*inductor or choke*: a circuit or part of a circuit that is designed to have a particular inductance.

The inductance of a circuit depends on its *size*, *shape*, and *number of turns* and the *magnetic properties* of the material enclosed by the circuit.

## 9 Transformers

*transformer*: device that transforms one voltage into another; consists of two coils—the **primary** (voltage supplied) and **secondary** (voltage delivered).

*Relation of voltage to winding turns for a transformer*

For an ideal transformer (with zero resistance),

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}.$$

By choosing the appropriate turns ration  $N_2/N_1$ , we may obtain any desired secondary voltage from a given primary voltage.

If  $V_2 > V_1$ , we have a *step-up* transformer; if  $V_2 < V_1$ , we have a *step-down* transformer. The  $V$ 's can be either both amplitudes or both rms values.

## 10 Magnetic Field Energy

Average power supplied by the current source during the small time interval  $\Delta t$  is:

$$P = V_{ab}i = Li \frac{\Delta i}{\Delta t}$$

*Energy stored in an inductor*

The energy stored by an inductor with inductance  $L$  carrying a current  $I$  is

$$U = \frac{1}{2}LI^2$$

*energy density*: the energy per unit volume, denoted by  $u$ , is

$$u = \frac{U}{V} = \frac{B^2}{2\mu_0}.$$

## 11 The R-L Circuit

**R-L circuit**: a circuit containing a resistor and an inductor. Due to inductance/resistance, there is no sudden change in current in an R-L circuit and thus we have current growth and current decay. Current growth is given by

$$i = \frac{\mathcal{E}}{R} \left( 1 - e^{-(R/L)t} \right);$$

current decay, by

$$i = I_0 e^{-(R/L)t}.$$

*time constant*:  $\tau = L/R$ .

## 12 The L-C Circuit

**L-C circuit**: a circuit containing an inductor and a capacitor leading to an oscillating current and charge called **electrical oscillation**. The angular frequency of the oscillatory behavior is given by

$$\omega = \sqrt{\frac{1}{LC}}$$

## 13 Links

MIT Video Demonstration of Lenz's Law