Physics 1214 — Chapter 22: Alternating Current — 02/25

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1 Phasors and Alternating Currents

alternating current: current that varies sinusoidally with time.
ac source: any device that supplies a sinusoidally varying potential difference v or current i. One function that can describe this voltage is $v = V\cos\omega t$, where:
voltage amplitude: V is the maximum potential difference.
instantaneous potential difference: v.
angular frequency: $\omega = 2\pi f$, where f is the frequency (the number of cycles per second).

The sinusoidal current i can be described by

$$\boxed{i = I\cos\omega t}$$

where
current amplitude: I is the maximum current.
phasors: rotating vector that rotates with frequency f and angular speed $\omega = 2\pi f$; the length of the phasor equals the *maximum current* I; the projection of the phasor onto the horizontal axis at time t equals the current i at that instant.
phasor diagrams: a diagram containing phasors!

> A phasor isn't a real physical quantity–just a way of describing quantities that vary sinusoidally in time.

root-mean-square (rms) current: I_{rms}

- square the instantaneous current, i
- calculate the average value of i^2
- take the square root of that average

rms values of sinusoidally varying current and voltage

> The rms current for a sinusoidal current with amplitude I is
>
> $$I_{rms} = \frac{I}{\sqrt{2}}.$$
>
> In the same way, the root-mean-square value of a sinusoidal voltage with amplitude (maximum value) V is
>
> $$V_{rms} = \frac{V}{\sqrt{2}}.$$

2 Resistance and Reactance

resistor in an ac circuit:

- $i = I\cos\omega t$ where maximum current amplitude is I.
- by Ohm's law (wrt points a and b), $v_R = iR = IR\cos\omega t$
- the maximum voltage $V_R = IR$
- then $v_R = V_R\cos\omega t$
- current i and voltage v are *in phase*

inductor in an ac circuit:

- pure inductor with self-inductance L and resistance zero
- $i = I \cos \omega t$
- $\mathcal{E} = -L\Delta i/\Delta t$
- then v_L (wrt points a and b), $v_L = L\Delta i/\Delta t$
- the voltage across the inductor at any instant is proportional to the *rate of change* of the current and they are *out of phase*

phase angle ϕ: where the voltage of one point wrt another is $v = V\cos(\omega t + \phi)$, the phase of the voltage relative to the current.

Pure inductor $\rightarrow \phi = 90^o$ and voltage leads the current
Pure resistor $\rightarrow \phi = 0$ and voltage and current are in phase

voltage amplitude (inductor)

$$\boxed{V_L = I\omega L}$$

Inductive reactance, X_L

The product of the inductance L and the angular frequency ω

$$X_L = \omega L.$$

capacitor in an ac circuit:

- capacitor with capacitance C
- $i = I \cos \omega t$
- voltage of a wrt b, $v_C = q/C$ where q is the charge on the plate
- the rate of change of v_C

$$\frac{\Delta v_C}{\Delta t} = \frac{1}{C}\frac{\Delta q}{\Delta t} = \frac{i}{C}$$

- then $\frac{\Delta v_C}{\Delta t} = \frac{1}{C} I \cos \omega t$ and with calculus $v_C = \frac{1}{\omega C}\sin \omega t$ and the voltage *lags* the current by 90^o

voltage amplitude (capacitor)

$$\boxed{V_C = \frac{I}{\omega C}}$$

Capacitive reactance

The inverse of the product of the angular frequency and the capacitance:

$$X_C = \frac{I}{\omega C}$$

Circuit element	Circuit quantity	Amplitude relation	Phase of v
Resistor	R	$V_R = IR$	in phase with i
Inductor	$X_L = \omega L$	$V_L = IX_L$	Leads i by $90°$
Capacitor	$X_C = 1/\omega C$	$V_C = IX_C$	Lags i by $90°$

3 The Series RLC Circuit

reactance

$$\boxed{X = X_L - X_C}$$

impedance (Z)

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X^2}}$$

Impedance of a series R-L-C Circuit

$$\begin{aligned} Z &= \sqrt{R^2 + Z^2} \\ &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{R^2 + [\omega L - (1/\omega X_C)]^2} \end{aligned}$$

Impedance is always a ratio of a voltage to a current; the SI unit of impedance is the ohm.

The impedance of any network as a ratio of the voltage amplitude to the current amplitude.

All the relations that have been developed in this chapter for a *R-L-C* circuit are still valid even if one of the circuit elements is missing.

We have been studying *steady-state* conditions–circuits which have been connected for a long time. Transient conditions are beyond the scope of this class.

4 Power in Alternating-Current Circuits

pure resistance circuit

- When a source with voltage amplitude V and instantaneous potential difference v supplies an instantaneous current i to an ac circuit, the instantaneous power p then $p = vi$.
- ***average power*** P: $P = \frac{1}{2}VI$ or

$$\boxed{P = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}} = V_{rms}I_{rms}}$$

pure inductor circuit

- $P = 0$

pure capacitor circuit

- $P = 0$

any ac circuit with combination of resistors, capacitors, and inductors

- instantaneous power $p = vi = [V\cos(\omega t + \phi)][I\cos\omega t]$
- average power

$$P = \frac{1}{2}VI\cos\phi = V_{rms}I_{rms}\cos\phi$$

power factor: $\cos\phi$

5 Series Resonance

resonance: a peaking of the current amplitude at a certain frequency in a constant voltage circuit.
resonance angular frequency: the angular frequency ω_0 at which the resonance peak occurs. The inductive and capacitive reactance are equal at the resonance angular frequency so

$$X_L = X_C, \quad \omega_0 L = \frac{1}{\omega_0 C}, \quad \omega_0 = \frac{1}{\sqrt{LC}}.$$

resonance frequency: $f_0 = \omega_0/2\pi$

Depending on the numerical values of R,L, and C, the voltages across L and C individually can be larger than that across R. At frequencies close to resonance, the voltages across L and C individually can be **much larger** than the source voltage!

6 Parallel Resonance

The instantaneous potential difference is the same for all three circuit elements and is equal to the source voltage $v = V\cos\omega t$, but the current is different in each of the three elements.

$$I_R = V/R$$

$$I_L = V/X_L = V/\omega L$$

$$I_C - V/X_C = V\omega C$$

and the impedance of the parallel combination

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + (\omega C - \frac{1}{\omega L})^2}$$

The total current in a parallel R-L-C circuit is minimum at resonance. If R is large, the impedance Z of the circuit near resonance is much larger than the individual reactances X_L and X_C and the individual currents in L and C can be **much larger** than the total current.