

## 1 Introduction to Electromagnetic Waves

*electromagnetic wave*: time-varying electric and magnetic field propagating through space from one region to another even when there is no matter in the intervening region (versus water waves, sound waves)

*Maxwell's Equations (one form)*:

$$\begin{aligned}\nabla \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} &= \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}\end{aligned}$$

## 2 Speed of an Electromagnetic Wave

*wave front*: the boundary of a wave

$$E = cB, \text{ only in SI units}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

*Characteristics of electromagnetic waves in vacuum*:

- the wave is transverse; both  $\vec{E}$  and  $\vec{B}$  are perpendicular to the direction of propagation of the wave and to each other
- there is a definite ratio between the magnitudes of  $\vec{E}$  and  $\vec{B}$ ;  $E = cB$
- the wave travels in vacuum with a definite and unchanging speed  $c$
- electromagnetic waves require no medium

## 3 The Electromagnetic Spectrum

*electromagnetic spectrum*: the broad spectrum of wavelengths covered by electromagnetic waves including radio, TV, and cellular phone transmission, microwaves, visible light, infrared and ultraviolet light, radiation, x rays and gamma rays.

*speed of electromagnetic waves in vacuum*:

$$c = \lambda f$$

where  $\lambda$  is the wavelength and  $f$  is the frequency of the wave

*monochromatic*: single-color light

## 4 Sinusoidal Waves

**plane wave:** at any instant the fields are uniform over any plane perpendicular to the direction of propagation. One form of the wave function for a transverse wave traveling to the right along a stretched string:

$$y(x, t) = A \sin(\omega t - kx)$$

where  $y$  is the transverse **displacement** from its equilibrium position at **time**  $t$  of a point with

coordinate  $x$  on the string where

$A$  is the maximum displacement or **amplitude** of the wave

$\omega$  is the **angular frequency** ( $2\pi$  times the **frequency**  $f$ )

$k$  is the **wave number** or *propagation constant* ( $2\pi/\lambda$ ) where

$\lambda$  is the **wavelength**

Where  $E$  and  $B$  are the instantaneous values of the electric and magnetic fields respectively, and  $E_{max}$  and  $B_{max}$  are the amplitudes of those fields

$$E = E_{max} \sin(\omega t - kx)$$

$$B = B_{max} \sin(\omega t - kx)$$

thus the sinusoidal oscillations of  $\vec{E}$  and  $\vec{B}$  are *in phase* and the amplitudes are related by

$$E_{max} = cB_{max}$$

**linearly polarized:** a wave whose  $\vec{E}$  field always lies along the same line.

## 5 Energy in Electromagnetic Waves

**energy density:** energy per unit volume.

### *Energy density in electric and magnetic fields:*

The energy density  $u$  (energy per unit volume) in a region of empty space where electric and magnetic fields are present is

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2.$$

The two field magnitudes are related by

$$B = \frac{E}{c} = \sqrt{\epsilon_0\mu_0}E.$$

Thus the energy density  $u$  can also be expressed as

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}(\sqrt{\epsilon_0\mu_0}E)^2 = \epsilon_0 E^2$$

and the energy density associated with the  $\vec{E}$  field is equal to the that of the  $\vec{B}$ .

**intensity:**  $I$ , the average power per unit area in an electromagnetic wave;  $I = S_{average}$ .

$$I = S_{av} = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_{max}^2 = \frac{1}{2} \epsilon_0 c E_{max}^2 = \frac{E_{max} B_{max}}{2\mu_0}.$$

**radiation pressure:** the force due to the absorption of an electromagnetic wave on a surface perpendicular to the direction of the propagation of the wave; the rate of change of the momentum  $p$  per unit area or  $I/c$  where  $c$  is the speed of light.

## 6 Nature of Light

“What’s in a name? that which we call a rose  
By any other name would smell as sweet;” Sir William Shakespeare

but can a rose be something else entirely?

**Light—both wave and particle**

- wave (continuous) best for understanding propagation (path)
- *photons* or *quanta* (discrete bundles of energy) best for understanding absorption by atoms and nuclei (energy)
- quantum electrodynamics: a comprehensive theory that includes both wave and particles properties

fundamental sources of all electromagnetic radiation (EMR) are electric charges in accelerated motion

all objects emit EMR as a result of thermal motion of their molecules (*thermal radiation*)

**wave front:** the locus of all adjacent points at which the phase of vibration of the wave is the same.

**ray:** an imaginary line along the direction of travel of the wave.

**geometric optics:** branch of optics using the ray behavior of light

**physical optics:** branch of optics using wave behavior

## 7 Reflection and Refraction

*reflection*: light that scattered off of the incident material.

*refraction*: light that is transmitted through the incident material.

*specular reflection*: reflection at a definite angle from a very smooth surface.

*diffuse reflection*: scattered reflection from a rough surface.

### *index of refraction*

The index of refraction of an optical material, denoted as  $n$ , is the ratio of the speed of light in vacuum ( $c$ ) to the speed of light in the material ( $v$ ):

$$n = \frac{c}{v}.$$

Light always travels more slowly in a material than in vacuum, so  $n$  for any material is always greater than one. For vacuum,  $n=1$  by definition.

### *Principles of geometric optics*

1. The incident, reflected, and refracted rays, and the normal to the surface, all lie in the same plane.
2. The angle of reflection  $\Theta_r$  is equal to the angle of incidence  $\Theta_a$  for all wavelengths and for any pair of substances;  $\Theta_r = \Theta_a$ . **Law of reflection.**
3. With qualifiers, the ratio of the sines of the angles  $\Theta_a$  and  $\Theta_b$ , where both angles are measured from the normal to the surface, is equal to the inverse ratio of the two indexes of refraction:

$$\frac{\sin \Theta_a}{\sin \Theta_b} = \frac{n_b}{n_a} \quad \text{or} \quad n_a \sin \Theta_a = n_b \sin \Theta_b.$$

**Law of refraction** or **Snell's law.**

The wavelength  $\lambda$  of the light in a material is less than its wavelength  $\lambda_0$  in a vacuum by the factor  $n$ :

$$\lambda = \frac{\lambda_0}{n}.$$

## 8 Total Internal Reflection

*critical angle*: the angle of incidence from which the refracted ray emerges tangent to the surface.

### *Total internal reflection*

When a ray traveling in a material  $a$  with index of refraction  $n_a$  reaches an interface with a material  $b$  having index  $n_b$ , where  $n_b < n_a$ , it is totally reflected back into material  $a$  if the angle incidence is greater than the critical angle given by

$$\sin \Theta_{critical} = \frac{n_b}{n_a}.$$

## 9 Dispersion

*dispersion*: the dependence of wave speed and index of refraction on wavelength

## 10 Polarization

*linearly polarized*: a wave with displacement in only one plane

*polarizing filter or polarizer*: a filter that polarizes a wave in a certain direction

*dichroism*: the selective absorption of one of the polarized components much more strongly than the other

*polarizing axis*: the axis in which the light is polarized parallel to

### *Light transmitted by polarizing filter*

When linearly polarized light strikes a polarizing filter with its axis at an angle  $\phi$  to the direction of polarization, the intensity of the transmitted light is

$$I = I_{max} \cos^2 \phi,$$

where  $I_{max}$  is the maximum intensity of the light transmitted (at  $\phi = 0$ ) and  $I$  is the amount transmitted at angle  $\phi$ . This is known as **Malus's Law**.

*plane of incidence*: the plane containing the incident and reflected rays and the normal to the surface.

*polarizing angle*: the angle for which  $\vec{E}$  is perpendicular to the plane of incidence (and parallel to the reflecting surface), thus the reflected light is linearly polarized perpendicular to the plane of incidence.

*Brewster's law*:

$$\frac{\sin \Theta_p}{\cos \Theta_p} = \tan \Theta_p = \frac{n_b}{n_a}$$

*photoelasticity*: the property of certain materials, when placed under mechanical stress, to have their index of refraction different for different planes of polarization.

## 11 Huygens Principle

### *Huygen's Principle*

Every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave.

## 12 Scattering of Light

*scattering*:

## 13 Links

Poster of EM spectrum pdf right click and download; extensive  
Berkeley EM scale pdf  
Mathematica Demonstrations: Electromagnetic Wave

### **EM radiation in the news:**

Supermassive Black holes and from Muse Supermassive Black hole

## 14 Greek

alpha	$\alpha$	A
beta	$\beta$	B
gamma	$\gamma$	$\Gamma$
delta	$\delta$	$\Delta$
epsilon	$\epsilon$ or $\varepsilon$	E
zeta	$\zeta$	Z
eta	$\eta$	H
theta	$\theta$ or $\vartheta$	$\Theta$
iota	$\iota$	I
kappa	$\kappa$	K
lambda	$\lambda$	$\Lambda$
mu	$\mu$	M
nu	$\nu$	N
xi	$\xi$	$\Xi$
omicron	$\omicron$	O
pi	$\pi$ or $\varpi$	$\Pi$
rho	$\rho$ or $\varrho$	P
sigma	$\sigma$ or $\varsigma$	$\Sigma$
tau	$\tau$	T
upsilon	$\upsilon$	$\Upsilon$
phi	$\phi$ or $\varphi$	$\Phi$
chi	$\chi$	X
psi	$\psi$	$\Psi$
omega	$\omega$	$\Omega$

## 15 Constants

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speed of light in a vacuum	$c = 2.99792458 \times 10^8 \text{ m s}^{-1}$
mass of the electron	$m_e = 9.1093826(16) \times 10^{-31} \text{ kg}$
mass of the proton	$m_p = 1.67262171(29) \times 10^{-27} \text{ kg}$
mass of the neutron	$m_n = 1.67492728(29) \times 10^{-27} \text{ kg}$
electric force constant or electrostatic constant	$k = 8.987551789 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ $k = \frac{1}{4\pi\epsilon_0}$
vacuum permittivity	$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$
fundamental unit of charge	$e = 1.602176565(35) \times 10^{-19} \text{ C}$
vacuum permeability	$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$

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