Math 1613 - Trigonometry

Exam #3 - 2010.10.07

Solutions

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1. Convert 215° to radian measure.

$$215^\circ \frac{\pi}{180^\circ} = \frac{215}{180}\pi = \frac{43}{36}\pi$$

2. Convert $\frac{15}{4}\pi$ radians to degree measure.

$$\frac{15}{4}\pi \frac{180^\circ}{\pi} = 675^\circ$$

3. Which is larger, 8 radians or 400°?

   8 radians is larger, consider:

$$8\frac{180^\circ}{\pi} = \frac{1440^\circ}{\pi} > 400^\circ$$

4. Find each of the following circular function values.

   a) $\sin\left(\frac{19}{6}\pi\right)$

$$\sin\left(\frac{19}{6}\pi\right) = \sin\left(2\pi + \frac{7}{6}\pi\right) = \sin\left(\frac{7}{6}\pi\right) = -\sin\left(\frac{1}{6}\pi\right) = -\frac{1}{2}$$

   b) $\csc\left(\frac{5}{3}\pi\right)$

$$\csc\left(\frac{5}{3}\pi\right) = \csc\left(2\pi - \frac{1}{3}\pi\right) = \csc\left(-\frac{1}{3}\pi\right) = -\csc\left(\frac{1}{3}\pi\right) = -\frac{2}{\sqrt{3}}$$

   c) $\tan\left(\frac{4}{3}\pi\right)$

$$\tan\left(\frac{4}{3}\pi\right) = \tan\left(\pi + \frac{1}{3}\pi\right) = \tan\left(\frac{1}{3}\pi\right) = \sqrt{3}$$

   d) $\cos\left(\frac{25}{4}\pi\right)$

$$\cos\left(\frac{25}{4}\pi\right) = \cos\left(6\pi + \frac{1}{4}\pi\right) = \cos\left(\frac{1}{4}\pi\right) = \frac{1}{\sqrt{2}}$$

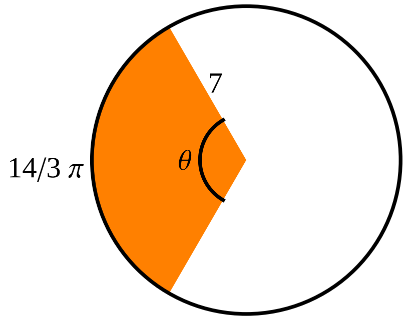
5. Determine the angular speed (in radians per hour) of the tip of a minute hand of a clock that is 8cm long.

$$\omega = \frac{\theta}{t} = \frac{2\pi}{1}\frac{\text{radians}}{\text{hour}}$$

6. Determine the linear speed (in cm per sec) of the tip of an hour hand of a clock that is 5cm long.

$$\begin{aligned} v = r\omega &= 5\text{ cm}\frac{2\pi\text{ rad}}{12\text{ hr}} \\ &= 5\text{ cm}\frac{2\pi\text{ rad}}{12\text{ hr}}\frac{1\text{ hr}}{60\text{ min}}\frac{1\text{ min}}{60\text{ sec}} \\ &= \frac{10\pi}{12\cdot 60^2}\frac{\text{cm}}{\text{sec}} \\ &= \frac{\pi}{4320}\frac{\text{cm}}{\text{sec}} \end{aligned}$$

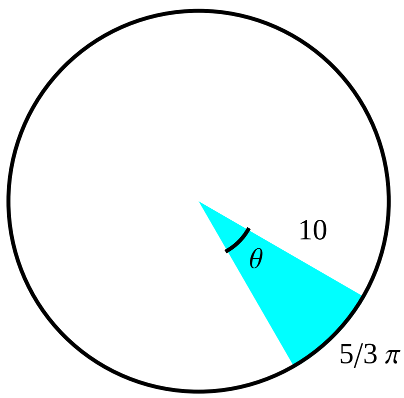
7. Find the area of the sector depicted in the following figure.



First we use $s = r\theta$ with $s = \frac{14}{3}\pi$ and $r = 7$ to get $\theta = \frac{2}{3}\pi$. Then we use $A = \frac{1}{2}r^2\theta$ with $r = 7$ and $\theta = \frac{2}{3}\pi$ to get

$$A = \frac{1}{2}7^2\frac{2}{3}\pi = \frac{49}{3}\pi$$

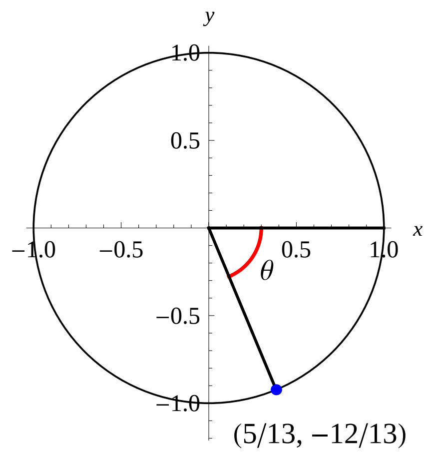
8. Find the measure of the angle given in the figure below.



Here we only have to use $s = r\theta$, with $s = \frac{5}{3}\pi$ and $r = 10$. This gives

$$\theta = \frac{s}{r} = \frac{\frac{5}{3}\pi}{10} = \frac{\pi}{6}$$

9. Evaluate the six circular functions for the value of θ depicted in the following figure.



Since the point lies on a circle of radius 1, we have that:

$$\cos(\theta) = x = \frac{5}{13}, \quad \sin(\theta) = y = -\frac{12}{13}$$

From these two, we also get that

$$\tan(\theta) = -\frac{12}{5}, \quad \sec(\theta) = \frac{13}{5}, \quad \csc(\theta) = -\frac{13}{12}, \quad \cot(\theta) = -\frac{5}{12}$$

10. Using the values of $\sin(\theta)$ and $\cos(\theta)$ from the previous problem, evaluate $\sin^2(\theta) + \cos^2(\theta)$.

$$\begin{aligned} \sin^2(\theta) + \cos^2(\theta) &= \left(\frac{5}{13}\right)^2 + \left(-\frac{12}{13}\right)^2 \\ &= \frac{25}{169} + \frac{144}{169} \\ &= \frac{169}{169} \\ &= 1 \end{aligned}$$

11. Find the exact value of θ in the given interval that has the given circular function value.

a) $\left[\frac{3}{2}\pi, 2\pi\right]; \quad \csc(\theta) = -\frac{2}{\sqrt{3}}$

First, we find a value of θ in the first quadrant such that $\csc(\theta) = \frac{2}{\sqrt{3}}$, or $\sin(\theta) = \frac{\sqrt{3}}{2}$. This happens when $\theta = \frac{\pi}{3}$. So using this as a reference angle we get that $\theta = 2\pi - \frac{\pi}{3} = \frac{5}{3}\pi$.

b) $[11\pi, 12\pi]; \quad \tan(\theta) = -\sqrt{3}$

First, note that $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$. Now we need an angle in the third or fourth quadrant, between $[11\pi, 12\pi]$, such that $\tan(\theta) = -\sqrt{3}$. Since tan is negative in the fourth quadrant, our angle should be $12\pi - \frac{\pi}{3} = \frac{35}{3}\pi$