Math 1613 - Trigonometry

Exam #4 - 2010.11.16

Name: _____

Pythagorean Identities:

$$\sin^2(\theta) + \cos^2(\theta) = 1 \qquad \tan^2(\theta) + 1 = \sec^2(\theta) \qquad 1 + \cot^2(\theta) = \csc^2(\theta)$$

Sum and Difference Identities:

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B) \qquad \cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B) \qquad \sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$\tan(A+B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)} \qquad \tan(A-B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$$

Double-Angle Identities:

$$\cos(2A) = \cos^2(A) - \sin^2(A) \qquad \cos(2A) = 1 - 2\sin^2(A)$$

$$\cos(2A) = 2\cos^2(A) - 1 \qquad \sin(2A) = 2\sin(A)\cos(A)$$

$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}$$

Product-To-Sum Identities:

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos(A)\sin(B) = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

Sum-To-Product Identities:

$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Verify each of the following identities.

1.

$$\frac{1}{\sec(t)-1}+\frac{1}{\sec(t)+1}=2\cot(t)\csc(t)$$

2.

$$\frac{\tan(x+y)-\tan(y)}{1+\tan(x+y)\tan(y)}=\tan(x)$$

3.

$$\sin\left(\frac{\pi}{6}+x\right)+\sin\left(\frac{\pi}{6}-x\right)=\sqrt{3}\cos(x)$$

4.

$$\sin(4x)=4\sin(x)\cos(x)-8\sin^3(x)\cos(x)$$

5.

$$\sin(2x) = \frac{2\tan(x)}{1+\tan^2(x)}$$

6.

$$\tan(\theta)\sin(2\theta) = 2 - 2\cos^2(\theta)$$

7. Find $\sin(y)$ given that $\cos(2y) = -\frac{1}{3}$, with $\frac{\pi}{2} < y < \pi$.

8. Find $\sin(x - y)$, $\cos(x - y)$ and what quadrant $x - y$ is located in, given that $\sin(x) = -\frac{1}{2}$, $\cos(y) = -\frac{2}{5}$ and x, y are angles located in quadrant III.