Math 1613 - Trigonometry

Final Exam - 2010.12.07

Name: ____________________________

Sum and Difference Identities:

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1+\tan(A)\tan(B)}$$
$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

Cofunction Identities:

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin(\theta), \quad \sin\left(\frac{\pi}{2}-\theta\right) = \cos(\theta), \quad \tan\left(\frac{\pi}{2}-\theta\right) = \cot(\theta)$$

Product-to-Sum and Sum-to-Product Identities:

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$
$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$
$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$
$$\cos(A)\sin(B) = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$
$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Double-Angle Identities:

$$\cos(2A) = \cos^2(A) - \sin^2(A) = 1 - 2\sin^2(A) = 2\cos^2(A) - 1$$
$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$$

Half-Angle Identities:

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}, \quad \cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{1+\cos(A)}}, \quad \tan\left(\frac{A}{2}\right) = \frac{\sin(A)}{1+\cos(A)}, \quad \tan\left(\frac{A}{2}\right) = \frac{1-\cos(A)}{\sin(A)}$$

Law of Sines:
In any triangle ABC with sides a, b and c,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}.$$

Law of Cosines:
In any triangle ABC with sides a, b and c,

$$a^2 = b^2 + c^2 - 2bc\cos(A), \quad b^2 = a^2 + c^2 - 2ac\cos(B), \quad c^2 = a^2 + b^2 - 2ab\cos(C).$$

Area of a Triangle:
In any triangle ABC with sides a, b and c, the area $\mathcal{A}$ is given by

$$\mathcal{A} = \frac{1}{2}bc\sin(A) = \frac{1}{2}ab\sin(C) = \frac{1}{2}ac\sin(B).$$

If we define the semiperimeter to be $s = \frac{1}{2}(a+b+c)$, then *Heron's formula* states

$$\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}.$$

1. For each of the four quadrants, state the sign of the three trig function cos, sin and tan.

2. State the exact values of the six trigonometric functions for the angle $\theta = \frac{11}{6}\pi$.

3. Convert 205° to radians.

4. Convert the radian measure $\frac{7}{9}\pi$ to degrees.

5. Find the exact value of $\cos\left(\frac{5}{12}\pi\right)$.

6. Express $\sec(\theta) - \sin(\theta)\tan(\theta)$ as a single function of θ

7. Graph the function $y = -1 + 2\cos\left(2x + \frac{\pi}{3}\right)$ over a two period interval. Be sure to indicate both the period and amplitude of the function on your graph.

8. Graph the function $y = 2 - 3\cot\left(x + \frac{\pi}{4}\right)$ over a two period interval. Be sure to state the period of the function on your graph.

9. Verify the identity

$$\frac{\sin(2x)}{2\sin(x)} = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$$

10. Find *all* solutions to the given equation for θ on the interval $[0, 2\pi)$:

$$\cos(2\theta) - \cos(\theta) = 0$$

11. Two ships leave a port at the same time, with the first sailing on a bearing of $32°$ at 10 knots, and the second on a bearing of $122°$ at 12 knots. How far apart are they after 3 hours? (Knots are nautical miles/hour)

12. Find the area enclosed in the triangular region which has the port and two ships as vertices at the 3 hour mark.

13. Find $\sin(y)$ given that $\cos(2y) = -\frac{1}{3}$, with $\frac{\pi}{2} < y < \pi$.

14. Using the *Law of Sines*, show that

$$\frac{\sin(a) - \sin(b)}{\sin(a) + \sin(b)} = \frac{a - b}{a + b}.$$