Math 1613 - Trigonometry

Quiz #26 - 2011.11.17

Solutions

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1. Give a geometric reason as to why De Moivre's Theorem is true. The identity from this theorem is given below:

$$\left[r(\cos(\theta) + i\ \sin(\theta)\right]^n = r^n\left[\cos(n\theta) + i\ \sin(n\theta)\right]$$

First, we remember that if we multiply two complex numbers together, we take the product of their magnitudes, and the resulting angle the product makes with the x-axis is the sum of the individual angles each complex number makes with the x-axis. If we take the two complex numbers to be the same, then the resulting magnitude will be the original magnitude squared and the angle will be twice the original angle. Apply this process n times, and you end up with a magnitude of r^n and an angle of $n\theta$, which is exactly De Moivre's Theorem.

2. Express the complex number $z = -3 - 3\sqrt{3}\,i$ in complex trigonometric form, using radian angle measure.

First, we compute the magnitude r, which is given by

$$r = \sqrt{(-3)^3 + (-3\sqrt{3})^2} = 6$$

Next we need to compute θ:

$$\tan(\theta) = \frac{-3\sqrt{3}}{-3} = \sqrt{3}$$

which gives $\theta = \frac{\pi}{3}$. However, this is only a reference angle since our complex number lies in the third quadrant. Thus, our angle is $\pi + \frac{\pi}{3} = \frac{4}{3}\pi$. So our number in complex trigonometric form is

$$z = 6\left[\cos\left(\frac{4}{3}\pi\right) + i\ \sin\left(\frac{4}{3}\pi\right)\right]$$