Math 1613 - Trigonometry
Final Exam

Name: _____

Instructions:

Please show all of your work. If you need more room than the problem allows, use a new plain white sheet of paper with the problem number printed at the top. You are allowed pencils, erasers, and a calculator, nothing else. Please show all work!

Problem Point Distribution:

1	2 pts	14	6 pts
2	2 pts	15	5 pts
3	5 pts	16	6 pts
4	4 pts	17	5 pts
5	4 pts	18	3 pts
6	10 pts	19	4 pts
7	12 pts	20	6 pts
8	5 pts	21	6 pts
9	5 pts	22	5 pts
10	7 pts	23	7 pts
11	6 pts	24	7 pts
12	6 pts	25	6 pts
13	6 pts	26	10 pts
		Total	150 pts

Sum and Difference Identities:

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1+\tan(A)\tan(B)}$$
$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

Cofunction Identities:

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin(\theta), \quad \sin\left(\frac{\pi}{2}-\theta\right) = \cos(\theta), \quad \tan\left(\frac{\pi}{2}-\theta\right) = \cot(\theta)$$

Product-to-Sum and Sum-to-Product Identities:

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$
$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$
$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$
$$\cos(A)\sin(B) = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$
$$\sin(A)+\sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A)-\sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos(A)+\cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A)-\cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Double-Angle Identities:

$$\cos(2A) = \cos^2(A) - \sin^2(A) = 1 - 2\sin^2(A) = 2\cos^2(A) - 1$$
$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$$

Half-Angle Identities:

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}, \quad \cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{1+\cos(A)}}, \quad \tan\left(\frac{A}{2}\right) = \frac{\sin(A)}{1+\cos(A)}, \quad \tan\left(\frac{A}{2}\right) = \frac{1-\cos(A)}{\sin(A)}$$

Law of Sines:
In any triangle ABC with sides a, b and c,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}.$$

Law of Cosines:
In any triangle ABC with sides a, b and c,

$$a^2 = b^2 + c^2 - 2bc\cos(A), \quad b^2 = a^2 + c^2 - 2ac\cos(B), \quad c^2 = a^2 + b^2 - 2ab\cos(C).$$

Area of a Triangle:
In any triangle ABC with sides a, b and c, the area $\mathcal{A}$ is given by

$$\mathcal{A} = \frac{1}{2}bc\sin(A) = \frac{1}{2}ab\sin(C) = \frac{1}{2}ac\sin(B).$$

If we define the semiperimeter to be $s = \frac{1}{2}(a+b+c)$, then *Heron's formula* states

$$\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}.$$

Length of an Arc of a Circle:
If r is the radius of the circle and θ is the angle of the arc, then the arclength, s, is $s = r\theta$.

Area of a Sector of a Circle:
If r is the radius of the circle and θ is the angle of the sector, then the area, A, of the sector is $A = \frac{\theta}{2}r^2$.

1. Convert $43°45'$ to decimal degrees.

2. Convert $42.35°$ to degrees, minutes and seconds.

3. Complete the following table:

θ deg	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
θ rad									
$\sin(\theta)$									
$\cos(\theta)$									
$\tan(\theta)$									

4. Solve for β in the following equation: $\sec(3\beta - 14°) = \csc(2\beta + 18°)$

5. Evaluate $\sin(1305°)$.

6. Sketch the graph of $f(x) = -2 + \frac{1}{3}\sin\left(2x - \frac{\pi}{6}\right)$ through two whole periods.

7. Sketch the graph of $g(x) = \frac{1}{2} + \sec\left(\frac{x}{3} + \frac{\pi}{4}\right)$ through two whole periods.

8. Find $\tan(\theta)$ given that $\sin(\theta) = \dfrac{2}{\sqrt{5}}$ with $\sec(\theta) < 0$.

9. If $\csc(\theta) = \dfrac{x+2}{x}$, find $\cot(\theta)$.

10. Find $\sin(s-t)$ if $\sin(s) = \dfrac{\sqrt{2}}{7}$ and $\sin(t) = \dfrac{\sqrt{3}}{8}$ with s and t both in the first quadrant.

11. Verify:

$$\frac{\sin(s+t)}{\cos(s)\cos(t)} = \tan(t) + \tan(s)$$

12. Verify:

$$\sin^2\left(\frac{x}{2}\right) = \frac{\tan(x) - \sin(x)}{2\tan(x)}$$

13. Verify:

$$\csc(x)\sin(2x) - \sec(x) = \cos(2x)\sec(x)$$

14. Find an exact value for the following expression:

$$\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(-\frac{3}{5}\right)\right)$$

15. Rewrite the following expression in terms of u, with $u > 0$ without trigonometric functions.

$$\sin\left(2\sec^{-1}\left(\frac{u}{2}\right)\right)$$

16. Solve for all $\theta \in [0, 2\pi)$ which satisfy the equation

$$\sqrt{2}\sin(3\theta) - 1 = 0$$

17. Solve for x in the equation $\cos^{-1}(x) = \sin^{-1}\left(\frac{1}{2}\right)$.

18. Write the product $(3 + 4\,i)(6 - 2\,i)(-1 + 2\,i)$ in standard form.

19. Convert $-2\sqrt{3} + 6\,i$ to $r\,\mathrm{cis}(\theta)$ form.

20. Find all values of z such that $z^4 = -2\sqrt{3} + 6\,i$.

21. If $z_1 = 13\sqrt{3}\,\mathrm{cis}(43^\circ)$ and $z_2 = 5\,\mathrm{cis}(123^\circ)$, compute both $z_1 z_2$ and $\dfrac{z_1}{z_2}$, expressing both answers in $r\,\mathrm{cis}(\theta)$ form.

Doctor Who
... a trigonometric approach....

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22. The Slitheen, whose homeworld is Raxacoricofallapatorius, are a rather tall bipedal species. To determine the height of a Slitheen, Rory, who is one inch less than 6 feet tall, notices that his shadow measures 2 feet 3 inch, whilst the Slitheens shadow measures 3 feet 1 inch. To the nearest inch, how tall is the Slitheen in question?

23. On the Dalek mothership, Emporer Dalek has a ramp up to his transparent cylindrical tank, situated below a giant dome, (naturally, the Dalek race prefers ramps to stairs). The ramp has a base length of 100 yards, and the angle of the incline is $28°$. How long is the inclined portion of the ramp, and how tall is the ramp at its end?

24. The Daleks have captured the TARDIS and wish to pull it up the ramp so that Emperor Dalek can gaze upon his fine trophy. Under the current gravitational force of the Dalek mothership, the TARDIS weighs 1500 kilograms. At any given point on the ramp, how much force (in kilograms) must be overcome to slide the TARDIS up the ramp?

25. If work is force times distance, how much work does it take to slide the TARDIS from the base of the ramp all the way to the top?

26. The Doctor is 128 yards from the TARDIS, at a bearing of 35° from his current position. A group of cybermen spot the doctor, and determine that he is 148 yards away at a bearing of 283° from their location. The Doctor asks Rose, who is located in the TARDIS, to determine the bearing and distance, from the TARDIS to the cybermen. What is her answer?