Math 1613 - Trigonometry

Final Exam - 2007.11.27

Due Date - 2007.12.11 - 8:00 A.M.

Name: _____

Instructions

Write each problem on its own page and be sure to show all your work. Compose your answers in a very concise and neat manner.

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1. Using the *Law of Sines*, show that

$$\frac{\sin(\alpha) - \sin(\beta)}{\sin(\alpha) + \sin(\beta)} = \frac{a - b}{a + b}.$$

2. Using the standard trigonometric identities, prove that

$$\cos(u) + \sin(u) = \sqrt{2}\cos\left(\frac{\pi}{4} - u\right).$$

3. Using the *Law of Cosines*, show that

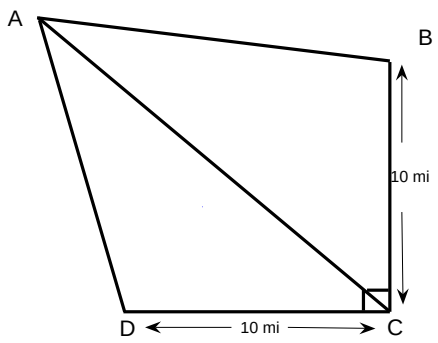
$$\cos(\alpha) + 1 = \frac{(b + c - a)(a + b + c)}{2bc}.$$

4. Graph each of the following functions. Also state the amplitude (when it exists), period and phase shift for each function. Be sure to incorporate this information into your graphs as well.

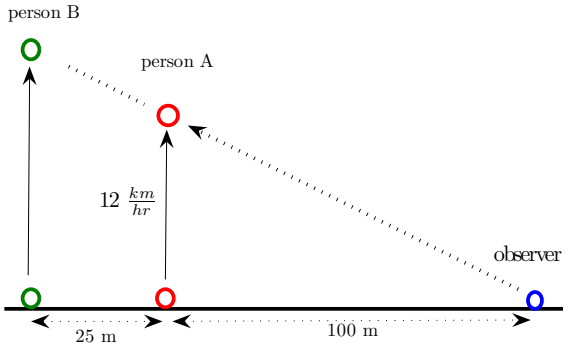
a) $f(x) = \frac{1}{3}\sin\left(\frac{x}{3} - \frac{\pi}{4}\right) + 4$

b) $g(x) = 3\tan\left(2x + \frac{5\pi}{6}\right) - 1$

5. A man always drives at the same speed. He makes it from A direct to C in 30 minutes; from A through B to C in 35 minutes; and from A through D to C in 40 minutes. How fast does he drive?



6. Three people lie along a line as depicted below. The person on the far right is the observer. Person A is 100 meters from the observer, and person B is 25 meters further down the line. If person A starts running at a constant rate of 12 $\frac{km}{hr}$, how fast does person B have to be running to stay directly behind person A in regards to the line of sight from the observer?



7. Prove the following identities:

a)
$$\sqrt{(3\cos(\theta) - 4\sin(\theta))^2 + (3\sin(\theta) + 4\cos(\theta))^2} = 5$$

b)
$$\frac{1}{\sin(\theta) + \cos(\theta)} + \frac{1}{\sin(\theta) - \cos(\theta)} = \frac{2\sin(\theta)}{\sin^4(\theta) - \cos^4(\theta)}$$

c)
$$\cos^4(\theta) = \frac{1}{8}\left(3 + 4\cos(2\theta) + \cos(4\theta)\right)$$

d)
$$\frac{\cos(3\theta) - \cos(\theta)}{\sin(\theta) - \sin(3\theta)} = \tan(2\theta)$$

8. Write $\cos\left(\theta - \frac{\pi}{6}\right)$ as a function of $\cos(\theta)$ and $\sin(\theta)$ only.

9. Solve the following equation, give your answers as exact values.

$$2\sin^2(\theta) - 2\cos^2(\theta) = 1$$

10. Compute $\left(-1 + \frac{1}{\sqrt{3}}i\right)^{15}$.

11. Find the seven seventh roots of the complex number $4\sqrt{3} - 4i$. Graph each of the seven roots in the complex plane as well.

12. Rewrite the following statement and sign your name to it:

"I hereby swear that all the work that appears on this written exam is completely my own, and I have not discussed any portion of this exam with any one."