Math 1613 - Trigonometry
Final Exam

Name: ____________________________

Instructions:

Please show all of your work. If you need more room than the problem allows, use a new plain white sheet of paper with the problem number printed at the top. You are allowed pencils, erasers, and a calculator, nothing else. Please show all work!

Problem Point Distribution:

1	2 pts	14	6 pts
2	2 pts	15	5 pts
3	5 pts	16	6 pts
4	4 pts	17	5 pts
5	4 pts	18	3 pts
6	10 pts	19	4 pts
7	12 pts	20	6 pts
8	5 pts	21	6 pts
9	5 pts	22	5 pts
10	7 pts	23	8 pts
11	6 pts	24	6 pts
12	6 pts	25	6 pts
13	6 pts	26	10 pts
		Total	150 pts

Sum and Difference Identities:

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1+\tan(A)\tan(B)}$$
$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

Cofunction Identities:

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin(\theta), \quad \sin\left(\frac{\pi}{2}-\theta\right) = \cos(\theta), \quad \tan\left(\frac{\pi}{2}-\theta\right) = \cot(\theta)$$

Product-to-Sum and Sum-to-Product Identities:

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$
$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$
$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$
$$\cos(A)\sin(B) = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$
$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Double-Angle Identities:

$$\cos(2A) = \cos^2(A) - \sin^2(A) = 1 - 2\sin^2(A) = 2\cos^2(A) - 1$$
$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$$

Half-Angle Identities:

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}, \quad \cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{1+\cos(A)}}, \quad \tan\left(\frac{A}{2}\right) = \frac{\sin(A)}{1+\cos(A)}, \quad \tan\left(\frac{A}{2}\right) = \frac{1-\cos(A)}{\sin(A)}$$

Law of Sines:
In any triangle ABC with sides a, b and c,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}.$$

Law of Cosines:
In any triangle ABC with sides a, b and c,

$$a^2 = b^2 + c^2 - 2bc\cos(A), \quad b^2 = a^2 + c^2 - 2ac\cos(B), \quad c^2 = a^2 + b^2 - 2ab\cos(C).$$

Area of a Triangle:
In any triangle ABC with sides a, b and c, the area $\mathcal{A}$ is given by

$$\mathcal{A} = \frac{1}{2}bc\sin(A) = \frac{1}{2}ab\sin(C) = \frac{1}{2}ac\sin(B).$$

If we define the semiperimeter to be $s = \frac{1}{2}(a+b+c)$, then *Heron's formula* states

$$\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}.$$

Length of an Arc of a Circle:
If r is the radius of the circle and θ is the angle of the arc, then the arclength, s, is $s = r\theta$.

Area of a Sector of a Circle:
If r is the radius of the circle and θ is the angle of the sector, then the area, A, of the sector is $A = \dfrac{\theta}{2}r^2$.

1. Convert $82°15'$ to decimal degrees.

2. Convert $42.35°$ to degrees, minutes and seconds.

3. Complete the following table:

θ deg	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
θ rad									
$\sin(\theta)$									
$\cos(\theta)$									
$\tan(\theta)$									

4. Solve for β in the following equation: $\sec(3\beta + 10°) = \csc(\beta + 8°)$

5. Evaluate $\cos(2280°)$.

6. Sketch the graph of $f(x) = -\dfrac{1}{2} + 3\cos\left(3x + \dfrac{\pi}{3}\right)$ through two whole periods.

7. Sketch the graph of $g(x) = 2 - \frac{1}{3}\csc\left(\frac{x}{2} + \pi\right)$ through two whole periods.

8. Find $\sin(\theta)$ given that $\tan(\theta) = -\frac{1}{\sqrt{2}}$ with $\sec(\theta) > 0$.

9. If $\sec(\theta) = \frac{x}{x-1}$, find $\tan(\theta)$.

10. Find $\cos(s-t)$ if $\sin(s) = \frac{\sqrt{5}}{7}$ and $\sin(t) = \frac{\sqrt{6}}{8}$ with s and t both in the first quadrant.

11. Verify:

$$\frac{\sin(s-t)}{\sin(t)} + \frac{\cos(s-t)}{\cos(t)} = \frac{\sin(s)}{\sin(t)\cos(t)}$$

12. Verify:

$$\frac{\cot(A) - \tan(A)}{\cot(A) + \tan(A)} = \cos(2A)$$

13. Verify:

$$\tan\left(\frac{\theta}{2}\right) = \csc(\theta) - \cot(\theta)$$

14. Find an exact value for the following expression:

$$\cos\left(\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{5}{13}\right)\right)$$

15. Rewrite the following expression in terms of u, with $u > 0$ without trigonometric functions.

$$\cot\left(\sin^{-1}\left(\frac{u}{\sqrt{u^2+2}}\right)\right)$$

16. Solve for all $\theta \in [0, 2\pi)$ which satisfy the equation

$$4\cos(2\theta) = 8\sin(\theta)\cos(\theta)$$

17. Solve for x in the equation $\sin^{-1}(x) = \tan^{-1}\left(\frac{3}{4}\right)$.

18. Write the product $(2+i)(2-i)(4+3\,i)$ in standard form.

19. Convert $4\sqrt{3} + 4\,i$ to $r\,\mathrm{cis}(\theta)$ form.

20. Find all values of z such that $z^5 = 2 - 2\sqrt{3}\,i$.

21. If $z_1 = 3\sqrt{2}\,\mathrm{cis}(47^\circ)$ and $z_2 = 2\,\mathrm{cis}(23^\circ)$, compute both $z_1 z_2$ and $\dfrac{z_1}{z_2}$, expressing both answers in $r\,\mathrm{cis}(\theta)$ form.

The Lord of the Rings
... a trigonometric approach....

---

22. The Dark Tower, the tallest tower of the fortress Barad-dûr, atop which the Eye of Sauron sees all, casts a shadow 1210 feet long at the same time that the shadow of Gollum, a creature whose slouched figure measures only 3.3 feet tall, casts a shadow 2.1 feet long. How tall is the Dark Tower?

23. Frodo needs to know the height of the wall surrounding Mordor. From a point on the ground he calculates the angle of elevation to the top of the wall to be $38°$. Frodo then moves back 50 feet, measures the new angle to the top of the wall, and finds it to be $36°$. How tall is the wall surrounding Mordor, and how far was Frodo from the wall when he made the second measurement?

24. Sauron, sensing Frodo's presence, decides to send out a patrol of Uruk-hai orcs in search of Frodo. The Black Gate of Mordor, which is quite large and heavy, is raised with the help of a single pulley. This pulley utilizes a wheel of radius 10 feet. Through what angle must the pulley wheel be rotated to raise the gate 20 feet off the ground to let the patrol through?

25. In the battle for Helms Deep, Saruman sends a battering ram loaded with explosives, pushed by a battalion of orcs, across a drawbridge. As the orcs begin crossing this drawbridge, the Rohirrim decide to use some trigonometry to help them out. Once the orcs are half way across the bridge, they decide to raise the bridge through an angle of $30°$, and then jam the pulley system so that it stays stuck at this level of inclination. If the ram weighs 8500 pounds, determine the force (in pounds) that the orcs must exert to stop the ram from sliding back off the drawbridge.

26. The distance, 'as the Nazgûl flies', from Mount Doom, located deep in the heart of Mordor, to the Shire, is 1560 miles. If one wishes to travel from Mount Doom to the Shire, but avoid Fangorn forest and the the tough terrain of mountains passes, one cannot travel 'as the Nazgûl flies'. Instead one should start off from Mount Doom on a bearing of 283° for 924 miles, then turn and continue for another 741 miles to reach the Shire. Find the bearing from the Shire to Mount Doom.