Math 2215 - Calculus 1

Homework #1

Assigned - 2011.01.12

Name: ______________________________

Textbook problems:

Section 1.1 - 1, 6, 11

Section 1.2 - 1, 2, 3, 7, 9, 15, 25, 33

Section 1.3 - 1, 2, 8, 10, 13, 26, 29, 31, 39, 40, 64, 65

Section 1.4 - 1, 7, 14, 22, 23, 25, 27, 33, 40, 50

Section 1.5 - 1, 4, 5, 14, 19, 23, 31, 35, 41, 43

Fun Problems:

1. In the theory of relativity, the mass of a particle with velocity v is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the mass of the particle at rest and c is the speed of light. What happens as $v \to c^-$ mathematically? Explain this result in terms of physical quantities and properties also.

2. Also in relativity, the Lorentz contraction formula

$$L = L_0\sqrt{1 - v^2/c^2}$$

expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest and c is the speed of light. Find the limit as $v \to c^-$ mathematically. Explain this result in terms of physical quantities and properties also.

3. A common equation used to determine gravitational time dilation is derived from the *Schwarzschild metric*, which describes spacetime in the vicinity of a non-rotating massive spherically-symmetric object. The equation is:

$$t_o = t_f\sqrt{1 - \frac{2GM}{rc^2}}$$

where t_o is the time perceived between events for person X near the object in question, and t_f is the time perceived be a person far away from the object in question (i.e. not under the influence of any massive object, including the one under consideration). Also, G is the gravitational constant, M is the mass of the object creating the gravitational field, r is the radial distance from person X to the center of the massive object, and c is the speed of light. Also, if we define the *Schwarzschild radius* as $r_0 = \frac{2GM}{c^2}$, the the time t_0 can be expressed

$$t_0 = t_f\sqrt{1 - \frac{r_0}{r}}.$$

(Note that if a mass collapses so that its surface lies at less than this radial coordinate r_0, then the object exists within a black hole.) As person X travels closer to the massive object (i.e. as $r \to r_0^+$) how do they perceive time with respect to the person far away? Also, how does the person far away perceive time with respect to person X as person X gets closer and close to the massive object?

4. Give an example of a function $f(x)$ such that $|f(x)|$ is continuous everywhere but $f(x)$ itself is discontinuous at at least one point.

5. If a and b are positive numbers, prove that the equation

$$\frac{a}{x^3 + 2x^2 - 1} + \frac{b}{x^3 + x - 2} = 0$$

has at least one solution in the interval $(-1, 1)$.

6. Find numbers a and b such that

$$\lim_{x \to 0} \frac{\sqrt{ax + b} - 2}{x} = 1$$

7. Define $f(x)$ as follows:

$$f(x) = \begin{cases} 0, & x \text{ is irrational} \\ x, & x \text{ is rational} \end{cases}$$

for what values of x is $f(x)$ continuous?

8. Construct a *Mathematica* function which performs a rudimentary bisection root-finding algorithm for locating a root of a function $f(x)$ on a specified closed interval $[a, b]$. You do not have to do any fancy graphics if you do not want.