Math 2215 - Calculus 1

Quiz #13 - 2011.04.20

Solutions

Consider the region bounded by the three functions $\{y = -x,\ y = x,\ y = \frac{1}{2}x - 1\}$. Set up the integrals (but do not evaluate) for both the washer and cylinder method for the volume of this region revolved about the line $y = 2$.

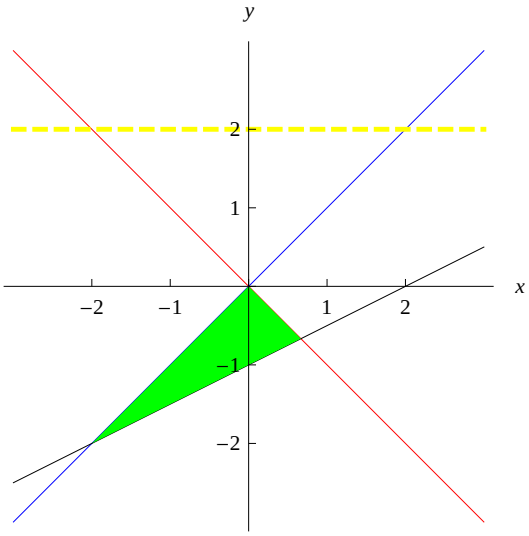

FIGURE 1. Region (green) to be revolved about the line $y = 2$.

By setting each line equal to each other line, we find that the intersection points of the three lines are $(-2, -2)$, $(0, 0)$, and $(2/3, -2/3)$. We first do the washer method, which would be a dx integral:

$$V_W = \int_{-2}^{0} \pi \left[\left(2 - \left(\frac{1}{2}x - 1 \right) \right)^2 - (2 - x)^2 \right] dx + \int_{0}^{2/3} \pi \left[\left(2 - \left(\frac{1}{2}x - 1 \right) \right)^2 - (2 - (-x))^2 \right] dx$$

The method of cylindrical shells will result in a dy integral. Note that $r(y) = 2 - y$ for all $-2 \leq y \leq 0$, and only $h(y)$ will change functions at $y = -2/3$:

$$V_{CS} = \int_{-2}^{-2/3} 2\,\pi\,(2 - y)\,((2y + 2) - y)\,dy + \int_{-2/3}^{0} 2\,\pi\,(2 - y)\,(-y - y)\,dy$$