

Math 2215 - Calculus 1

Quiz #1 - 2011.01.24

Solutions

Compute the following limits.

1. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} &= \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2} \\ &= \lim_{x \rightarrow 2} \frac{x - 2}{x - 2} (x + 2) \\ &= \lim_{x \rightarrow 2} \frac{x - 2}{x - 2} \lim_{x \rightarrow 2} (x + 2) \\ &= 1 \cdot 4 = 4\end{aligned}$$

2. $\lim_{x \rightarrow 2^-} \frac{x^2 + 4}{x - 2}$

Notice that the numerator is always positive. Thus, all we need to do is figure out what happens in the denominator. As $x \rightarrow 2$, the denominator goes to zero, and in particular, from the left, the denominator goes to zero and is negative. Thus

$$\lim_{x \rightarrow 2^-} \frac{x^2 + 4}{x - 2} = -\infty$$

3. $\lim_{x \rightarrow \infty} \frac{x^2 + 4x + 5}{3x^2 - 2x - 2}$

Since the degree of the numerator is equal to that of the denominator, we simply take the ratio of the leading coefficients.

$$\lim_{x \rightarrow \infty} \frac{x^2 + 4x + 5}{3x^2 - 2x - 2} = \frac{1}{3}$$