

Math 2215 - Calculus 1

Quiz #2 - 2011.01.25

Solutions

1. State the Intermediate Value Theorem.

Answers will vary, but here are the key ingredients:

Let $f(x)$ be a continuous function on the interval $[a, b]$. If d lies between the values of $f(a)$ and $f(b)$, then there is a $c \in [a, b]$ such that $f(c) = d$.

2. Use the Intermediate Value Theorem to prove that $f(x) = x - \cos(x)$ has at least one positive root.

First, note that $f(0) = -1$, and that $f(x)$ is continuous for all real numbers. In particular, note that if $a > 1$, then $f(a) > 0$ since $\cos(x) < 1$ for all x and thus $a - \cos(a) > 0$. Applying the Intermediate Value Theorem using the interval $[0, a]$, where $a > 1$ proves that there is a $c \in [0, a]$ such that $f(c) = 0$.