

Math 2215 - Calculus 1

Quiz #9 - 2011.03.04 Solutions

Consider the function $f(x) = \sqrt[5]{x}$.

1. Compute the tangent line to $f(x)$ at $x = -32$.

First we need $f'(x)$:

$$f'(x) = \frac{1}{5}x^{-4/5} = \frac{1}{5} \frac{1}{(\sqrt[5]{x})^4}$$

Then

$$\begin{aligned} L(x) &= f(-32) + f'(-32)(x + 32) \\ &= -2 + \frac{1}{5 \cdot 16}(x + 32) \\ &= -2 + \frac{1}{80}(x + 32) \end{aligned}$$

2. Use your answer to problem 1 to approximate $\sqrt[5]{-31.7}$ as a fraction of two integers (i.e. as a rational number).

We now simply compute $L(-31.7)$:

$$\begin{aligned} L(-31.7) &= -2 + \frac{1}{80}(-31.7 + 32) \\ &= -2 + \frac{1}{80}(0.3) \\ &= -2 + \frac{1}{80} \frac{3}{10} \\ &= -\frac{1600}{800} + \frac{3}{800} \\ &= -\frac{1597}{800} \end{aligned}$$

3. Using your knowledge about the graph of $f(x) = \sqrt[5]{x}$, will the real value of $\sqrt[5]{-31.7}$ be larger or smaller than your approximation?

Since the tangent line lies below the actual curve for $x < 0$, the approximation is smaller, i.e. $-\frac{1597}{800} < \sqrt[5]{-31.7}$. To verify (which you cannot do in class), we approximate each as a decimal:

$$-\frac{1597}{800} = -1.996250000 < \sqrt[5]{-31.7} \approx -1.996235858$$