Math 2215 - Calculus 1

Final Exam - 2011.04.25

Due Date - 2011.05.09 at 10:00 AM

Name: ___________________________

Instructions:

Please show all of your work, allow plenty of space between each problem, or part of problem. You do not have to start each problem on a separate sheet of paper. The only ways that you can get help on any of these problems is to look through books, look through notes, look at old course material on my website, or talk to me. Nothing else is allowed. Do not use decimal approximations for any numbers, and all problems in this exam do not require the use of a calculator, but you can use your calculator to verify your answers.

Problem Point Distribution:

1 a)	5pts	3 b)	6pts	7	15pts	9 b)	7pts	11 e)	7pts
1 b)	5pts	3 c)	7pts	8 a)	3pts	9 c)	7pts	11 f)	7pts
1 c)	5pts	3 d)	6pts	8 b)	5pts	9 d)	7pts	11 g)	7pts
1 d)	5pts	3 e)	6pts	8 c)	3pts	9 e)	7pts	12 a)	7pts
1 e)	5pts	3 f)	7pts	8 d)	5pts	9 f)	7pts	12 b)	13pts
1 f)	5pts	4 a)	5pts	8 e)	5pts	10	10pts	13 a)	5pts
1 g)	5pts	4 b)	5pts	8 f)	5pts	11 a)	5pts	13 b)	20pts
1 h)	5pts	4 c)	10pts	8 g)	4pts	11 b)	7pts	14)	10pts
2	15pts	5	10pts	8 h)	10pts	11 c)	7pts		
3 a)	6pts	6	15pts	9 a)	7pts	11 d)	7pts		

1. Compute the following limits:

(a) $\lim_{x \to 0} \frac{\tan(x^3)}{x^2}$

(b) $\lim_{x \to -1} \frac{x^2(x-2)}{(x-1)(x+1)^2}$

(c) $\lim_{x \to \infty} \frac{2x-1}{\sqrt{3x^2-4x}}$

(d) $\lim_{x \to -\infty} \frac{2x-1}{\sqrt{3x^2-4x}}$

(e) $\lim_{x \to -1^+} \frac{x^2(x-2)}{x^2-1}$

(f) $\lim_{x \to -1^-} \frac{x^2(x-2)}{x^2-1}$

(g) $\lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \left[\left(\frac{2k}{n}\right)^3 + \left(\frac{3k}{n}\right)^2 - \frac{5k}{n} \right]$

(h) $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n} \sqrt{\frac{k}{n}}$

2. Find the value of b such that the following limit exists and is finite. Also find the corresponding limit for the value of b that you found. You may assume that $a > 0$.

$$\lim_{x\to\infty} \sqrt{ax^2 + 4x - 2} - (bx + 4)$$

3. Compute the following derivatives. In general, you do not have to simplify your answers, however please simplify your answer to part (c).

(a) $\dfrac{d}{dz}\dfrac{\sin(\cos(z))}{\cos(z^2) + z^2 + z}$

(b) $\dfrac{d}{dw}\displaystyle\int_{-w^2}^{\sin(w)} \frac{1}{3}x^3 - \frac{1}{2}x^2 + x - 1 \, dx$

(c) $\dfrac{d}{dx}\left[\dfrac{x^2+1}{x^2-1} - \dfrac{1}{x-1}\left(\dfrac{2}{x+1} - 1\right) - \dfrac{x}{x-1}\right]$

(d) $\dfrac{d}{dt}\sqrt[5]{5t - \dfrac{3}{t^4}}$

(e) $\dfrac{d}{dr}F(\sin^2(r) - \cos(r))G(12r^2 - 4r + 1)$

(f) $\dfrac{d}{d\theta}\sin(\tan(\cos(\sec(\csc(\cot(\theta^2))))))$

4. Consider the curve given by $y^2 = x^3 - 27x + 90$. Use it to answer all of the following questions.

(a) Find $\frac{dy}{dx}$.

(b) Find the points where $y'(x) = 0$.

(c) Show that all of the tangent lines at the points on the curve with horizontal tangent lines intersect the curve at points with rational coordinates.

5. Find $\frac{dw}{dt}$ given the relation

$$\sin(3w - t) + \cos(3t - w) = 1.$$

6. Show that $\sqrt{a^2 + b} \approx a + \frac{b}{2a}$ if b is small and $a > 0$. Use this to estimate $\sqrt{98}$.

7. Show that $f(x) = x + \frac{x}{x^2+1}$ has precisely one root.

8. Answer parts (a) through (h) for the following function:

$$g(t) = \frac{t^2 + 2t - 1}{t^2 + 2t + 3}$$

(a) State the domain

(b) State all horizontal or vertical asymptotes

(c) Locate all roots of the equation

(d) Find local maximum and minimum values

(e) State the intervals of increase and decrease

(f) Find the intervals of concavity

(g) Locate any inflection points

(h) Graph the function using your answers to parts a) - g).

9. Compute the following integrals:

(a) $\displaystyle\int \csc(x) - x\cot(x)\csc(x)\, dx$

(b) $\displaystyle\int_0^{2\pi} \sin^2(t)\sin(\sin(t)))\, dt$

(c) $\displaystyle\int z\sqrt{2z+1}\, dz$

(d) $\displaystyle\int \sqrt{x^3+1}x^5\, dx$

(e) $\displaystyle\int_0^{2\pi} \sin^2(w)\, dw$

(f) $\displaystyle\int (\sin(t)+\cos(t))^2\, dt$

10. Find the average value of the function $c(x) = \sqrt{16 - x^2}$ on the interval $[-4, 4]$.

11. Consider the following functions:

$$f_1(x) = x^2 - 1,\ \ f_2(x) = -(x-1)^2 + 4,\ \ f_3(x) = 2x + 2,\ \ f_4(x) = 2x - 1$$

(a) Sketch the region bounded by the four curves. Be sure to include all intersection points.

(b) Find the area bounded by all four curves using a dx integral.

(c) Find the area bounded by all four curves using a dy integral.

(d) Find the volume of the solid found by rotating the region about the line $y = -2$.

(e) Find the volume of the solid found by rotating the region about the line $y = 4$.

(f) Find the volume of the solid found by rotating the region about the line $x = -1$.

(g) Find the volume of the solid found by rotating the region about the line $x = 3$.

12. Consider the function $f(x) = \frac{1}{2}x^2$, for $0 \leq f(x) \leq 5$.

(a) Find the volume of the bowl generated by rotating $f(x)$ about the y-axis over the specified region.

(b) If the bowl is being filled with water at a constant rate 3 cubic units per second, how fast will the water level in the bowl be rising when the water is 4 units deep?

13. Consider the ellipse given by

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1,$$

where $\alpha, \beta > 0$, and the point $P(\alpha + 2, 0)$ on the positive x-axis. Answer the following questions.

(a) Graph several ellipses, changing values of α and β, along with the point P.

(b) Find a relationship (an inequality) involving α and β such that the point on the ellipse, corresponding to the maximum distance between the ellipse and the point P, does not occur at the point $(-\alpha, 0)$, rather at two points symmetric about the x-axis instead.

14. Compute the arclength of the curve given by $y^{\frac{2}{3}} + x^{\frac{2}{3}} = 1$, depicted below.