Math 2283 - Introduction to Logic

Quiz #10 - 2010.10.15

Solutions

Let us define our universe to be the set of days of the week, which are symbolically given by

$$\bigvee = \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$$

We next define the following relations on the universe $\bigvee$:

$x \ \mathcal{C} \ y$ iff x is the day of the week which immediately precedes the day y chronologically. For instance Tuesday $\mathcal{C}$ Wednesday, **as well as** Saturday $\mathcal{C}$ Sunday.

$x \ \mathcal{L} \ y$ iff x is a day of the week whose name precedes the name of day y alphabetically. For instance Tuesday $\mathcal{L}$ Wednesday, since the word "Tuesday" precedes the word "Wednesday" alphabetically. In a similar fashion, we also have that Friday $\mathcal{L}$ Wednesday.

1. Determine if the relation $\mathcal{C}$ is any of: reflexive, irreflexive, symmetric, asymmetric, transitive, and connected.

First, notice that no day of the week can satisfy that it appears chronologically before itself. Therefore, not only is $\mathcal{C}$ not reflexive, it is irreflexive.

Furthermore, if a day of the week comes before another day of the week chronologically, it is never true vice versa. We thus get that $\mathcal{C}$ is not symmetric, but is in fact asymmetric.

The relation is $\mathcal{C}$ is not transitive, since there will be a day in between the two days in question, thus the first not immediately preceding the second.

By a similar argument, $\mathcal{C}$ is not connected. For instance, one cannot relate Tuesday and Friday by either Tuesday $\mathcal{C}$ Friday or Friday $\mathcal{C}$ Tuesday.

2. Determine if the relation $\mathcal{L}$ is any of: reflexive, irreflexive, symmetric, asymmetric, transitive, and connected.

First, notice that no day of the week can satisfy that it appears alphabetically before itself. Therefore, not only is $\mathcal{L}$ not reflexive, it is irreflexive.

Furthermore, if a day of the week comes before another day of the week alphabetically, it is never true vice versa. We thus get that $\mathcal{L}$ is not symmetric, but is in fact asymmetric.

The relation is $\mathcal{L}$ is transitive, since if word x comes before word y, and word y comes before word z, then word x comes before word z alphabetically. This is a property of lexicographical ordering.

$\mathcal{L}$ is connected! If I take any two distinct days of the week, one must come first alphabetically, hence $x \ \mathcal{L} \ y \vee y \ \mathcal{L} \ x$ for any two distinct days of the week x and y, which is the definition of a relation being connected.