

Math 2283 - Introduction to Logic

Quiz #11 - 2010.11.01 Solutions

There is one error in the following proof of Theorem XI. Correct it!

Axiom I and Definition I are given as:

Axiom I. $K \subset K$.

Definition I. $K = L \leftrightarrow K \subset L \wedge L \subset K$.

We now prove Theorem XI:

Theorem XI. $K = K$.

Proof: An instance of Definition I with $K = L$ yields:

$$(1) \quad K = K \leftrightarrow K \subset K \wedge K \subset K$$

An instance of the logical law $p \wedge p \rightarrow p$ with $p = K \subset K$ is

$$(2) \quad K \subset K \wedge K \subset K \rightarrow K \subset K$$

Substituting (2) into (1) gives

$$(3) \quad K = K \leftrightarrow K \subset K$$

Applying mROD with Axiom I and (3) gives Theorem XI.

You cannot substitute (2) into (1) unless you use the logical law $p \wedge p \leftrightarrow p$ instead of $p \wedge p \rightarrow p$. So the proof should look as follows instead:

Proof: An instance of Definition I with $K = L$ yields:

$$(4) \quad K = K \leftrightarrow K \subset K \wedge K \subset K$$

An instance of the logical law $p \wedge p \rightarrow p$ with $p = K \subset K$ is

$$(5) \quad K \subset K \wedge K \subset K \leftrightarrow K \subset K$$

Substituting (5) into (4) gives

$$(6) \quad K = K \leftrightarrow K \subset K$$

Applying mROD with Axiom I and (6) gives Theorem XI.