

Math 2283 - Introduction to Logic

Quiz #13 - 2010.11.22

Solutions

Remember that the binary operation O is right monotonic in the class K with respect to the two-termed relation R if for any $x, y, z \in K$, we have:

$$yRz \longrightarrow (xOy)R(xOz)$$

If we define K to be the set of real numbers, and the relations A and B as was done in problem 3 of the current chapter:

$$xAy = x, \quad xBy = y,$$

then given the following relations, determine whether or not the property of right monotony is satisfied.

1. Relation R is $<$, with the operation A .

So we must ask if the following holds for all x, y , and z :

$$y < z \longrightarrow (xAy) < (xAz),$$

and we have that $xAy = xAz = x$, so the above statement is

$$y < z \longrightarrow x < x,$$

which does not hold true for all x, y , and z .

2. Relation R is $<$, with the operation B .

So we must ask if the following holds for all x, y , and z :

$$y < z \longrightarrow (xBy) < (xBz),$$

and we have that $xBy = y$ and $xBz = z$, so the above statement is

$$y < z \longrightarrow y < z,$$

which holds true for all x, y , and z .

3. Relation R is \leq , with the operation A .

So we must ask if the following holds for all x, y , and z :

$$y \leq z \longrightarrow (xAy) \leq (xAz),$$

and we have that $xAy = xAz = x$, so the above statement is

$$y \leq z \longrightarrow x \leq x.$$

Since the consequent is always true, we the statement is true.

4. Relation R is \leq , with the operation B .

This holds true by the exact same argument as 2.