Math 2283 - Introduction to Logic

Quiz #9 - 2010.10.13

Solutions

Remember that the relative product of the relations R and S, denoted R/S, is defined as

$$xR/Sy \longleftrightarrow \underset{z}{E}\, xRz \wedge zSy$$

Consider the following relations:

$$xRy \longleftrightarrow x + 2y = 4, \quad xSy \longleftrightarrow x + y > 0$$

Is the relative product relation R/S reflexive? Explain your answer fully. You may assume that the domain of discourse is the real numbers.

To prove that R/S is reflexive, we need to show that $\underset{x}{A}\, xR/Sx$:

$$\begin{aligned} xR/Sx &\longleftrightarrow \underset{z}{E}\, xRz \wedge zRx \\ &\longleftrightarrow \underset{z}{E}\, x + 2z = 4 \wedge z + x > 0 \end{aligned}$$

So the question becomes, is $\underset{z}{E}\, x + 2z = 4 \wedge z + x > 0$ true for all $x \in \mathbb{R}$? Solving for x in $x + 2z = 4$ gives $x = 4 - 2z$. Plugging this into $z + x > 0$ gives $-z + 4 > 0$, which we can solve for in terms of z; $z < 4$. However, if $z < 4$, this will not always allow the relation xRz to be satisfied. For instance, if $x = -6$, then $x + 2z = 4$ becomes $z = 5$ which implies that $\sim z < 4$.