Math 3283 - Foundations of Math

Final Exam - 2012.05.08

Name: ______________________________

Short Answer:

1. What three properties must a relation have to be considered an equivalence relation?

2. State the definition of a cartesian product.

3. How does one prove a biconditional statement?

4.Under what conditions can you prove a universally quantified statement with a constructive proof?

5. State the definition of a function being injective.

6. State which two properties are required of a function for it to be invertible.

7. There are two equivalent statements to "The set A is countable", give one of them.

8. Construct a truth table for the statement $(p \to q) \wedge \sim p \to \sim q$.

9. Determine if the following argument is valid:

$$\begin{array}{l} (x = y) \to (y = x) \\ (x = y) \wedge (y = z) \to (x = z) \\ \underline{(x = z) \wedge (y = z) \qquad\qquad} \\ \therefore\ x = y \end{array}$$

10. Let $U = \mathbb{R}$. Determine which of the following quantified statements are true.

(a) $\forall x, y,\ (x + y^2 > 1)$

(b) $\forall x, \exists y,\ (x + y^2 > 1)$

(c) $\exists y, \forall x,\ (x + y^2 > 1)$

(d) $\forall y, \exists x,\ (x + y^2 > 1)$

(e) $\exists x, \forall y,\ (x + y^2 > 1)$

(f) $\exists x, y,\ (x + y^2 > 1)$

11. Prove that if $f : A \to B$ is onto, and $g : B \to C$ is onto, then $g \circ f : A \to C$ is onto.

12. Write the negation of the following quantified statement *without using the symbol* '$\to$':

$$\forall y, \exists x, (f(a) < y < f(b)) \to [(a < x < b) \wedge f(x) = y]$$

13. Prove that for any sets A,B and C in a domain of discourse U, $\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$.

14. Prove that if n and m are odd integers, then nm is odd.

15. Prove that an integer n is odd iff $3n + 7$ is even.

16. Prove that for every positive integer n, $n^5 - n$ is divisible by 5.

17. Consider the relation R on $\mathbb{N}$ defined by xRy iff x and y are relatively prime. Determine if the relation R is and of the following: reflexive, symmetric, antisymmetric or transitive.

18. Let $A_1, A_2, \ldots, A_n$ be n sets. Prove $\chi_{A_1 \cap A_2 \cap \cdots \cap A_n} = \displaystyle\prod_{i=1}^{n} \chi_{A_i}$.

19. Compute the following infinite union, with proof: $\displaystyle\bigcup_{n=1}^{\infty} \left[3 + \frac{1}{n}, 5 + \frac{1}{n}\right)$