Math 3283 - Foundations

Midterm - 2012.02.23

Name: _____

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1. Determine which of the following are statements:

   (a) Today is Monday.

   (b) x^2 is positive.

   (c) $\exists x \in \mathbb{R}\, x^2 > 0$

   (d) $\exists x \forall y \in \mathbb{R}\, x^2 \leq y$

2. Construct a truth table for the statement $(p \wedge q) \rightarrow (\sim q \vee r)$.

3. The following four statements can be paired into two equivalencies. Determine the equivalencies.

   (a) $(p_1 \vee p_2) \rightarrow q$ (b) $(p_1 \rightarrow q) \wedge (p_2 \rightarrow q)$ (c) $(p_1 \wedge p_2) \rightarrow q$ (d) $(p_1 \rightarrow q) \vee (p_2 \rightarrow q)$

4. Determine if the following argument is valid.

   $p \rightarrow (q \vee r)$
   $p \vee r$
   $\underline{\sim r}$
   $\therefore q$

5. Determine if each of the following statements are true or false.

   (a) $\exists x \in \mathbb{C}\, (4x^8 - 12x^6 + 7x^5 + 2x^3 - 4x + 3 = 0)$

   (b) $\exists x \in \mathbb{W} \forall y, z \in \mathbb{W}\, (x \leq y \cdot z)$

   (c) $\exists! x \in \mathbb{Z} \forall y \in \mathbb{Z}\, (x \mid y)$

6. Negate the following statement: $\forall x \exists y\, (p(x, y) \vee \sim q(x))$

7. Prove or disprove: Let n and m be integers, then $n \mid 2m \rightarrow n \mid m$.

8. Prove or disprove: Let n and m be integers, then nm odd $\leftrightarrow$ n odd and m odd.

9. Prove or disprove: Let n and m be integers, then nm even $\leftrightarrow$ n even and m even.

10. Prove by induction that for any positive integer n:

$$[(p_1 \vee p_2 \vee \cdots \vee p_n) \rightarrow q] \Leftrightarrow [(p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \wedge \cdots (p_n \rightarrow q)]$$