Math 4133 - Linear Algebra

Homework #6

Assigned - 2011.03.28

Name: ______________________________

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1. Is it possible to find two subspaces, $\mathbb{S}_1$ and $\mathbb{S}_2$ of a vector space $\mathbb{V}$ such that $\mathbb{S}_1 \cup \mathbb{S}_2$ is a subspace of $\mathbb{V}$? Here, the assumptions you are allowed to make are that $\mathbb{S}_1$ and $\mathbb{S}_2$ are not equal, are not nonempty, and one is not a proper subset of the other.

2. Let $\mathbb{S}_1$ be the subspace of $\mathbb{R}^n$ corresponding to solutions to the homogeneous line system $A\overrightarrow{x} = \overrightarrow{0}$, and let $\mathbb{S}_2$ be the subspace of $\mathbb{R}^n$ corresponding to solutions to the homogeneous line system $B\overrightarrow{x} = \overrightarrow{0}$. Find an equation whose solution is the subspace $\mathbb{S}_1 \cap \mathbb{S}_2$.

3. Consider the following two subspace of $\mathbb{R}^4$:
$\mathbb{V}_1 = \{\, a\,\langle 1, 2, -1, 0\rangle + b\,\langle 2, -3, 2, 2\rangle \mid a, b \in \mathbb{R}\}$
$\mathbb{V}_2 = \{\, a\,\langle 2, 1, 1, 1\rangle + b\,\langle 3, 4, -5, 6\rangle + c\,\langle 10, -29, 18, 14\rangle \mid a, b, c \in \mathbb{R}\}$
Construct a basis for $\mathbb{V}_1 \cap \mathbb{V}_2$ and also for $\mathbb{V}_1 + \mathbb{V}_2$.

4. Using the vector subspaces from problem 3, compute the following:

   (a) $\mathbb{V}_1^{\perp}$

   (b) $\mathbb{V}_2^{\perp}$

   (c) $(\mathbb{V}_1 + \mathbb{V}_2)^{\perp}$

   (d) $(\mathbb{V}_1 \cap \mathbb{V}_2)^{\perp}$

5. Prove or disprove the following based on your answers to problem 4:

   (a) $(\mathbb{V}_1 + \mathbb{V}_2)^{\perp} = \mathbb{V}_1^{\perp} + \mathbb{V}_2^{\perp}$

   (b) $(\mathbb{V}_1 \cap \mathbb{V}_2)^{\perp} = \mathbb{V}_1^{\perp} \cup \mathbb{V}_2^{\perp}$