Math 4133 - Linear Algebra

Homework #7

Assigned - 2011.04.04

Name: ______________________________

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1. Construct an orthonormal basis for the subspace of $\mathbb{R}^3$ corresponding to the plane $3x - 5y + 6z = 0$.

2. Project the vector $\langle -1, 2, 3\rangle$ onto the subspace of $\mathbb{R}^3$ corresponding to the plane $3x - 5y + 6z = 0$.

3. Using problems 1 and 2, construct an orthonormal basis for $\mathbb{R}^3$ where two vectors line in the plane $3x-5y+6z = 0$.

4. Using the vectors from problem 3, construct the dot product matrix W.

5. Create an orthonormal set of vectors from the set

$$\{\langle -1+i, 3+2i, 6+5i, 3\rangle, \langle 2-i, 2+3i, 6-7i, 4i\rangle, \langle 0, 2+3i, 6, -1+i\rangle\}$$

6. Consider the set of all continuously and infinitely differentiable functions on the interval $[-1, 1]$ (denoted $\mathbb{C}^{\infty}_{[-1,1]}$). We know that if $f(x) \in \mathbb{C}^{\infty}_{[-1,1]}$, then $f(x)$ has a Maclaurin series expansion, i.e.,

$$f(x) = \sum_{k=1}^{\infty} a_k\, x^k$$

Thus, we can consider $\mathbf{B} = \left\{1, x, x^2, \ldots, x^n, \ldots\right\}$ to be a basis for $\mathbb{C}^{\infty}_{[-1,1]}$. For $f(x), g(x) \in \mathbb{C}^{\infty}_{[-1,1]}$, the "dot product" in this vector space is given by

$$f(x) \cdot g(x) = \int_{-1}^{1} f(x)g(x)\, dx$$

(a) Using the definition of the dot product given above, project the function $f(x) = \cos(\pi\, x)$ onto the functions $\left\{1, x, x^2, x^3, x^4\right\}$.

(b) It should be clear that the set $\mathbf{B}$ is NOT orthogonal (hence not orthonormal). Compute the first 5 orthogonal vectors functions corresponding to the first 5 basis functions of $\mathbf{B}$ (which are $\left\{1, x, x^2, x^3, x^4\right\}$).

(c) Normalize the functions from part (b) to create an orthonormal set.

(d) It would make sense that the projection of $f(x) = \cos(\pi\, x)$ onto the set in part (a) should be the same as the projection on to the set constructed in part (c), since they span the same subspace of $\mathbb{C}^{\infty}_{[-1,1]}$. Verify that this is indeed true.

(e) If you were to construct an entire orthonormal set from $\mathbf{B}$, you would end up with an orthonormal basis for $\mathbb{C}^{\infty}_{[-1,1]}$. The orthonormal basis form of the polynomial set $\mathbf{B}$ has a very specific name. See if you can find our more about this set by searching the Internet and looking in textbooks.