

Math 4213 - Complex Analysis

Midterm - 2012.02.27

Name: _____

- Express the $(2 - 3i)\overline{(3 + 4i)}$ in standard form.
- Sketch the region D defined by $D = \{z \mid \operatorname{Re}(z) < 0 \text{ and } |z - 1 + i| < 9\}$
- Compute $\operatorname{Arg}(z)$ if $z = (1 - i)^{15}$.
- Express the function $f(z) = \bar{z}e^z$ as $f(x + iy) = u(x, y) + iv(x, y)$.
- Find all values ω such that $\omega^{10} = 2 - 2i$.
- Consider the function $v(x, y) = 4xy - 3x - 1$.
 - Verify that $v(x, y)$ is harmonic on \mathbb{C}
 - Find the harmonic conjugate $u(x, y)$ to $v(x, y)$
 - Find a function $f(z)$ such that $f(z) = u(x, y) + iv(x, y)$. Write your answer in terms of z only, and simplify.
- Find all *real* numbers c such that the following limit exists. Please express your answer in interval notation.
$$\lim_{n \rightarrow \infty} \left(\frac{2}{c} - i \frac{3}{2c} \right)^n$$
- Find the set of points z for which the following power series is convergent:
$$g(z) = \sum_{n=2}^{\infty} \frac{(n-i)(n+1)}{(n-1)(2n-i)} (z-2+i)^n$$
- Find all values of z such that $e^z = 1 - i$.
- Find all values of z such that $\operatorname{Log}(z) = 1 - i\frac{\pi}{2}$.
- Evaluate the integral $\int_{\mathcal{C}} z^3 \cos(z) + z^2 \sin(z) dz$, where \mathcal{C} is the upper 1/2 of the circle of radius 7, centered at $z_0 = 0$ and oriented positively.
- Consider the function $f(z) = \frac{1}{z^4 - 1}$. Compute $\int_{\mathcal{C}} f(z) dz$ where \mathcal{C} is the positively oriented ellipse centered at $z_0 = i/2$ and semi-major axis of length 2 in the x -direction and semi-minor axis of length 1 in the y -direction.