

# Math 4213 - Complex Analysis

## Quiz #10 - 2012.02.06

### Solutions

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1. 1. How is a power series and its radius of convergence related?

A power series is only valid on its radius of convergence. There are three options:

- (1) Radius of convergence is good only at a single point  $\alpha$ .
- (2) Radius of convergence is good on a disc of radius  $r$ , centered at  $\alpha$ . This may or may not include points on the boundary.
- (3) Radius of convergence is  $\infty$ .

2. Find an analytic function  $f(z) = u(x, y) + i v(x, y)$  if  $v(x, y) = e^y \sin(x)$ .

If we assume analyticity, then we know that  $u_x = v_y$  and  $u_y = -v_x$ , so

$$v_y = e^y \sin(x) = u_x$$

So

$$\begin{aligned} u(x, y) &= \int u_x dx + G(y) \\ &= -e^y \cos(x) + G(y) \end{aligned}$$

To find  $G(y)$ , we compute  $u_y$ :

$$u_y = -e^y \cos(x) + G'(y) = -v_x = -e^y \sin(x)$$

so  $G'(y) = 0$ , and  $G(y)$  is a constant, which we set to 0. Thus

$$u(x, y) = -e^y \cos(x)$$

and

$$f(z) = -e^y \cos(x) + i e^y \sin(x)$$