

Math 4213 - Complex Analysis

Quiz #11 - 2012.02.08

Solutions

1. Why do we need to define a principal value for the logarithmic function?

The reason for this is that the exponential function is not one-to-one, thus its inverse cannot be defined unless we restrict ourselves to a specific range. This range becomes the principal domain for the principal logarithmic function.

2. Find a value for partial sum

$$S_n = \sum_{k=0}^n \left(\frac{1}{k+1+i} - \frac{1}{k+i} \right)$$

Since this is a finite sum, we break up as follows:

$$S_n = \sum_{k=0}^n \frac{1}{k+1+i} - \sum_{k=0}^n \frac{1}{k+i}$$

and we can reorder the first series as

$$S_n = \sum_{k=1}^{n+1} \frac{1}{k+i} - \sum_{k=0}^n \frac{1}{k+i}$$

Thus

$$\begin{aligned} S_n &= \left(\frac{1}{n+1+i} + \sum_{k=1}^n \frac{1}{k+i} \right) - \left(\frac{1}{i} + \sum_{k=1}^n \frac{1}{k+i} \right) \\ &= \frac{1}{n+1+i} - \frac{1}{i} + \sum_{k=1}^n \frac{1}{k+i} - \sum_{k=1}^n \frac{1}{k+i} \\ &= \frac{1}{n+1+i} + i \end{aligned}$$