

Math 4213 - Complex Analysis

Quiz #12 - 2012.02.15

Solutions

1. If we have a complex-valued function $f(z)$, and a contour C which is parametrized as $\gamma(t)$ for $t \in [a, b]$, how can we compute $\int_C f(z) dz$?

We simply set $z = \gamma(t)$, and apply the chain rule:

$$\int_C f(z) dz = \int_a^b f(\gamma(t))\gamma'(t) dt$$

2. Find two values z_1 and z_2 such that $e^{z_1} = e^{z_2} = \frac{3}{\sqrt{2}} - \frac{3}{\sqrt{2}}i$.

$$\begin{aligned} e^z &= 3 \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) \\ &= 3 \left(\cos\left(-\frac{\pi}{4} + 2\pi k\right) + i \sin\left(-\frac{\pi}{4} + 2\pi k\right) \right) \end{aligned}$$

So we have that $x = \ln(3)$ and $y = -\frac{\pi}{4} + 2\pi k$ for arbitrary k .