

# Math 4213 - Complex Analysis

## Quiz #13 - 2012.02.22

### Solutions

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1. Compute  $\int_{\mathcal{C}} z^2 - 2iz \, dz$ , where  $\mathcal{C}$  is the line connecting the points  $z_0 = 2 + i$  and  $z_1 = -1 - 2i$ .

We need to parameterize the curve  $\mathcal{C}$ , which we do with  $\gamma(t) = (1-t)z_0 + tz_1$ ,  $t \in [0, 1]$ , with  $\gamma'(t) = z_1 - z_0$ . So

$$\begin{aligned}\int_{\mathcal{C}} z^2 - 2iz \, dz &= \int_0^1 (\gamma^2(t) - 2i\gamma(t)) \gamma'(t) \, dt \\ &= \int_0^1 (18it^2 - 12it - 12t + 5)(-3 - 3i) \, dt \\ &= 3 - 9i\end{aligned}$$

Also, by Theorem 6.9, we can simply perform the following definite integral (after checking the hypotheses of the Theorem hold):

$$\begin{aligned}\int_{\mathcal{C}} z^2 - 2iz \, dz &= \int_{2+i}^{-1-2i} z^2 - 2iz \, dz \\ &= \left. \frac{1}{3}z^3 - iz^2 \right|_{2+i}^{-1-2i} \\ &= 3 - 9i\end{aligned}$$