

Math 4213 - Complex Analysis

Quiz #16 - 2012.03.12

Solutions

1. What is the difference between $\int_{-\infty}^{\infty} f(z) dz$ and P.V. $\int_{-\infty}^{\infty} f(z) dz$? Hint: Remember how we defined the first in Calculus 1 as a sum of limits.

From calculus, we have that

$$\int_{-\infty}^{\infty} f(z) dz = \lim_{A \rightarrow -\infty} \int_A^0 f(z) dz + \lim_{B \rightarrow \infty} \int_0^B f(z) dz$$

Note that A and B do not have to be the same variable, and thus, these are two independent limits. To compute the Cauchy principal value integral, we do let $A = B$, thus

$$\text{P.V.} \int_{-\infty}^{\infty} f(z) dz = \lim_{A \rightarrow \infty} \int_{-A}^A f(z) dz$$

As a simple example, note that P.V. $\int_{-\infty}^{\infty} z dz = 0$, whereas $\int_{-\infty}^{\infty} z dz$ does not exist.

2. Locate all of the poles of the following function and determine their order.

$$f(z) = \frac{\sin(z)}{z(z^2 + 1)^2}$$

The function $f(z)$ has a removable singularity at $z = 0$, and poles of order 2 at $z = \pm i$.