

Math 4213 - Complex Analysis

Quiz #7 - 2012.01.30

Solutions

1. Under what conditions is a complex valued function $f(z) = u(x, y) + i v(x, y)$ harmonic?

The function $f(z)$ must be analytic.

2. Show that $\lim_{z \rightarrow 0} \frac{\bar{z}}{z}$ does not exist.

To show that the limit does not exist, we simply try to compute the limit from at least two different directions with different results. For instance, if we let $z = r e^{i \theta_0}$ for a fixed $\theta_0 \in (-\pi, \pi]$. Then

$$\begin{aligned} \lim_{z \rightarrow 0} \frac{\bar{z}}{z} &= \lim_{r \rightarrow 0} \frac{\overline{r e^{i \theta_0}}}{r e^{i \theta_0}} \\ &= \lim_{r \rightarrow 0} \frac{r e^{-i \theta_0}}{r e^{i \theta_0}} \\ &= \frac{e^{-i \theta_0}}{e^{i \theta_0}} \cdot \lim_{r \rightarrow 0} \frac{r}{r} \\ &= e^{-i 2 \theta_0} \cdot \lim_{r \rightarrow 0} \frac{r}{r} \\ &= e^{-i 2 \theta_0} \end{aligned}$$

Thus, there are an infinite number of limits, one for each $\theta_0 \in (-\pi, \pi]$, so the limit itself does not exist.