

# Math 4213 - Complex Analysis

## Quiz #9 - 2012.02.03

### Solutions

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1. What is the difference between the supremum and limit supremum of a sequence  $\{z_n\}$ ?

The supremum must be a value which is greater than or equal to every element of the sequence, whilst the limit supremum is simply the limit of supremums of the tail of the sequence. That is

$$\limsup_{n \rightarrow \infty} z_n = \lim_{n \rightarrow \infty} \left( \sup_{m \geq n} z_m \right)$$

2. Determine if the function  $\phi(x, y) = e^y \sin(x)$  is harmonic.

We simply need to determine if  $\phi_{xx}(x, y) + \phi_{yy}(x, y) = 0$ . So

$$\phi_x(x, y) = e^y \cos(x), \phi_{xx}(x, y) = -e^y \sin(x)$$

$$\phi_y(x, y) = e^y \sin(x), \phi_{yy}(x, y) = e^y \sin(x)$$

Clearly,  $\phi_{xx}(x, y) + \phi_{yy}(x, y) = 0$ , so  $\phi(x, y)$  is harmonic.