

Math 4973 - Dynamical Systems

Homework #1

Assigned - 2011.06.06

Name: _____

1. Consider the doubling function $D(x) = 2x \bmod 1$, for $x \in [0, 1)$.
 - (a) For each of the initial starting values of the form $x_0 = \frac{1}{k}$ for $1 \leq k \leq 10$, list all values on each orbit.
 - (b) Classify each of the ten orbits found in part (a).
 - (c) Compute the orbit of any initial starting value of the form $x_0 = \frac{1}{2^n}$ for $n \in \mathbb{N}$.
 - (d) What do you think will happen to the orbit corresponding to an irrational initial starting value? Attempt to justify your answer.
 - (e) Graph $D(x)$.
2. Consider the composition of $D(x)$ with itself, i.e. $D^2(x) = (D \circ D)(x)$.
 - (a) Give an explicit formula for $D^2(x)$.
 - (b) Graph $D^2(x)$.
 - (c) Find all fixed points to $D^2(x)$, i.e. all points such that $x = D^2(x)$.
 - (d) For the points from part (c), compute the orbits under $D(x)$. What do you notice?
3. Consider the composition of $D(x)$ with itself n times, $D^n(x)$.
 - (a) Graph $D^n(x)$ for $n = 3, 4, 5$ and then explain what the graph of $D^n(x)$ should look like for arbitrary positive integers n .
 - (b) Find all the fixed points for $D^3(x)$, $D^4(x)$ and $D^5(x)$. How many fixed points are there for each of these functions?
 - (c) Discuss the possible number of fixed points for $D^n(x)$. What does this say about the orbits of these fixed points of $D^n(x)$ under $D(x)$?