Math 4973 - Dynamical Systems

Homework #4

Assigned - 2011.06.27

Name: ________________________________

Consider the following four functions, all of which have a parameter c:

(a) $F_c(x) = x^2 + c$, (b) $G_c(x) = c\,x\,(1-x)$, (c) $H_c(x) = c\,(2x^2 - 4x + 1)$, (d) $K_c(x) = e^{-8x^2} + c$

You may wish to download the Mathematica notebook BifFunctions.nb from the website.

1. Find the unique critical point for each of the given functions.

2. Using the critical points as your x_0, construct bifurcation diagrams for each function on the given regions for the parameter c:

   (a) $F_c(x)$ for $c \in [-2, 0]$

   (b) $G_c(x)$ for $c \in [1, 4]$

   (c) $H_c(x)$ for $c \in [-1.618, -0.860]$

   (d) $K_c(x)$ for $c \in [-1, 1]$

3. For each of the bifurcation diagrams from problem 2, find the values c_n which correspond to the map having a superstable orbit of period 2^n for $0 \le n \le 7$. Try to be as precise as possible (to at least 5 decimal place accuracy).

4. Using your answer from problem 3, compute $f_k = \frac{c_k - c_{k+1}}{c_{k+1} - c_{k+2}}$ for $0 \le k \le 5$. Do any of the f_k's appear to tend to the same value as k increases?