Math 1303 - Math in the Liberal Arts

Homework #1 - 2008.01.11

Due Date - 2008.01.18

Solutions

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1. Compute the next three terms in each of the following sequences by using inductive reasoning. Be sure to explain your reasoning.

a) 0, 1, 1, 2, 3, 5, 8, 13, 21 ...

This is the well known Fibonacci sequence, where you add the two previous numbers to get the next. Thus, 13 + 21 = 34, 21 + 34 = 55 and 55 + 34 = 89.

b) 1, 1, 2, 2, 3, 4, 4, 8, 5, 16, 6 ...

Starting with the first number, all the odd entries in the sequence are simply increasing by 1. The even index numbers are multiplied by 2. So we should expect the next three terms to be 32, 7, and 64.

c) 1, 2, 4, 6, 10, 14, 22, 26, 34 ...

Notice that after the first entry, one has that the sequence is simply the sequential prime numbers multiplied by 2. The next three prime numbers after 17 are 19, 23 and 29. Therefore, the next three numbers in the sequence are 38, 46 and 58.

d) 1, 11, 21, 1211, 111221, 312211, 13112221, 1113213211 ...

This sequence is a little bit trickier. Notice that the first entry has 1 one(1), the second has 2 ones(1), the third has 1 two(2) and 1 one(1) etc... The pattern is describing the number of each integer as they appear in the number. Hence, the last number in the sequence has 3 ones(1), 1 three(3), 1 two(2), 1 one,(1) 1 three(3), 1 two(2) and 2 ones(1). The number is thus given by 31131211131221. The same can be done for the number just determined. There is 1 three(3), 2 ones(1), 1 three (3), 1 one(1), 1 two(2), 3 ones(1), 1 three(3), 1 one(1), 2 twos(2) and 1 one(1). This gives the number 13211311123113112211. Doing this once more gives the number 1113122113311213211321222 1.

e) 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23 ...

Notice that the only natural numbers missing from this sequence are the perfect squares (i.e. 1, 4, 9, and 16). Thus, the next three entries should be 24, 26, and 27.

f) 11, 13, 17, 25, 32, 37, 47, 58, 71, 79, 95, 109, 119 ...

The next entry is obtained by by adding the previous entry to the sum of its digits. I.e. 13 = 11 + 1 + 1, 17 = 13+1+3 etc... The next term in the sequence would be given by 119+1+1+9 = 130, then 130+1+3+0 = 134 and finally 134 + 1 + 3 + 4 = 142.

2. Give two examples of estimation that you may have used recently and be sure to work out all the details. You may not use any example or problem from the book!
Answers will vary...

3. Give both a correct and incorrect example of inductive reasoning which does not involve mathematics.
Answers will vary...