

Math 1303 - Math in the Liberal Arts
Homework #3 - 2008.01.25
Due Date - 2008.02.01
Solutions

1. Express each repeating decimal as a quotient of two integers.

a) $1.7\overline{78}$

$$1.7\overline{78} = 1.77 + 0.00\overline{78} = \frac{177}{100} + 0.00\overline{78}$$

Setting $n = 0.00\overline{78}$, we get that $10n = 0.08\overline{78}$ and thus $9n = 0.08$, or $n = \frac{0.08}{9} = \frac{8}{900}$. So we now have

$$1.7\overline{78} = \frac{177}{100} + \frac{8}{900} = \frac{1601}{900}.$$

b) $1.\overline{778}$

$$1.\overline{778} = 1.7 + 0.0\overline{778} = \frac{17}{10} + 0.0\overline{778}$$

Setting $n = 0.0\overline{778}$, we get that $100n = 7.8\overline{778}$ and thus $99n = 7.8$, or $n = \frac{7.8}{99} = \frac{78}{990}$. So we now have

$$1.\overline{778} = \frac{17}{10} + \frac{78}{990} = \frac{1761}{990} = \frac{587}{330}.$$

c) $1.\overline{7\overline{78}}$

$$1.\overline{7\overline{78}} = 1. + 0.\overline{7\overline{78}}$$

Setting $n = 0.\overline{7\overline{78}}$, we get that $1000n = 778.\overline{7\overline{78}}$ and thus $999n = 778$, or $n = \frac{778}{999}$. So we now have

$$1.\overline{7\overline{78}} = 1 + \frac{778}{999} = \frac{1777}{999}.$$

2. Simplify the following numbers (if possible).

a) $\sqrt{5}(\sqrt{10} + 3\sqrt{15})$

$$\sqrt{5}(\sqrt{10} + 3\sqrt{15}) = 5\sqrt{2} + 15\sqrt{3}$$

b) $(3 + \sqrt{7})\sqrt{21}$

$$(3 + \sqrt{7})\sqrt{21} = 3\sqrt{21} + 7\sqrt{3}$$

c) $(\sqrt{5} - 4)(\sqrt{5} + 4)$

$$(3 + \sqrt{7})\sqrt{21} = 3\sqrt{21} + 7\sqrt{3} = -11$$

3. Rationalize the following fractions.

a) $\frac{3+\sqrt{2}}{\sqrt{7}}$

$$\frac{3+\sqrt{2}}{\sqrt{7}} = \frac{3+\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}+\sqrt{14}}{7}$$

b) $\frac{3+2\sqrt{5}}{\sqrt{7}+3}$

$$\frac{3+2\sqrt{5}}{\sqrt{7}+3} = \frac{3+2\sqrt{5}}{\sqrt{7}+3} \cdot \frac{\sqrt{7}-3}{\sqrt{7}-3} = \frac{(3+2\sqrt{5})(\sqrt{7}-3)}{(\sqrt{7}+3)(\sqrt{7}-3)} = \frac{3\sqrt{7}+2\sqrt{35}-9-6\sqrt{5}}{-2}$$

c) $\frac{4\sqrt{3}-6\sqrt{2}}{\sqrt{7}+\sqrt{3}}$

$$\frac{4\sqrt{3}-6\sqrt{2}}{\sqrt{7}+\sqrt{3}} = \frac{4\sqrt{3}-6\sqrt{2}}{\sqrt{7}+\sqrt{3}} \cdot \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{(4\sqrt{3}-6\sqrt{2})(\sqrt{7}-\sqrt{3})}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} = \frac{4\sqrt{21}-12-6\sqrt{14}+6\sqrt{6}}{4}$$