

Math 1303 - Math in the Liberal Arts

Homework #4 - 2008.02.01

Due Date - 2008.02.08

Solutions

1. Find at LEAST 5 constants that are known which are usually represented in scientific notation. Give a brief description of each constant and give the value. As an example, the distance that light travels in a second is usually measured in meters per second. The speed of light is approximately 3×10^8 meters per second.

Answers will vary...

2. Determine if the following sequences are arithmetic, geometric or neither. If the sequence is arithmetic or geometric, write an expression for the general or n th term of the sequence.

a) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots$

The answer is neither....

b) $4, -5, -14, -23, -32, -41, -50, \dots$

This is an arithmetic sequence with $d = -9$ and $a_1 = 4$. So $a_n = 4 + (n - 1)(-9)$.

c) $10, \frac{5}{2}, \frac{5}{8}, \frac{5}{32}, \frac{5}{128}, \frac{5}{512}, \frac{5}{2048}, \dots$

This is a geometric sequence with $a_n = 10 \cdot \frac{1}{4}^{n-1}$.

d) $-\frac{1}{3}, -\frac{1}{12}, \frac{1}{6}, \frac{5}{12}, \frac{2}{3}, \frac{11}{12}, \frac{7}{6}, \dots$

This is an arithmetic sequence with $d = \frac{1}{4}$ and $a_1 = -\frac{1}{3}$. So $a_n = -\frac{1}{3} + (n - 1)\frac{1}{4}$.

e) $10.0, 5.5, 1.0, -2.5, -8.0, -11.5, -17.0, \dots$

The answer is neither....

f) $2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots$

This is a geometric sequence with $a_n = 2 \cdot \frac{1}{2}^{n-1}$.

3. Consider the arithmetic sequence 1, 4, 7, 10, 13, 16, 19, 22, 25, 29, . . .

a) Find the sum of the first 1242 terms in the sequence.

First we write the general formula for the n th term in the sequence: $a_n = 1 + (n - 1)3$. This gives $a_{1242} = 1 + 1241 \cdot 3 = 3724$. the formula for the sum of the first 1242 terms is

$$s_{1242} = \frac{1242(1 + 3724)}{2} = 2313225.$$

b) Find the sum of the 123rd through 1242 terms in the sequence.

This may seem difficult, however, notice that we want the following sum:

$$a_{123} + a_{124} + \cdots + a_{1241} + a_{1242}$$

But what we can do is write it as follows:

$$a_1 + a_2 + \cdots + a_{122} + a_{123} + a_{124} + \cdots + a_{1241} + a_{1242} - (a_1 + a_2 + \cdots + a_{122})$$

which is $s_{1242} - s_{122}$. We have computed s_{1242} in the previous problem, so we simply compute s_{122} .

$$s_{122} = \frac{122(1 + 364)}{2} = 22265.$$

So our sum is $s_{1242} - s_{122} = 2313225 - 22265 = 2290960$.