Math 1513 - College Algebra

Quiz #10 - 2008.11.06

Solutions

Factor the polynomial $f(x) = x^5 - 2x^4 - x + 2$ into the product of 5 linear terms.

By the rational roots theorem, we know that the roots could potentially be ± 1 and ± 2. Plugging in these values of x shows that $x = 1$, $x = -1$ and $x = 2$ are roots. So we only have two left! So far we have the following:

$$x^5 - 2x^4 - x + 2 = (x-1) \cdot (x+1) \cdot (x-2) \cdot (\text{ a quadratic })$$

So now we do long division $x^5 - 2x^4 - x + 2$ of by $(x-1) \cdot (x+1) \cdot (x-2)$, which is really $x^3 - 2x^2 - x + 2$ after we multiply the trhree linear terms together.

$$\begin{array}{r|l} & \qquad\qquad\qquad\quad x^2 \qquad\quad + 1 \\ \hline x^3 - 2x^2 - x + 2 & x^5 - 2x^4 \qquad\qquad\qquad - x + 2 \\ & -x^5 + 2x^4 + x^3 - 2x^2 \\ & \qquad\qquad\quad x^3 - 2x^2 - x + 2 \\ & \qquad\qquad -x^3 + 2x^2 + x - 2 \\ & \qquad\qquad\qquad\qquad\qquad\qquad 0 \end{array}$$

So now we have

$$x^5 - 2x^4 - x + 2 = (x-1) \cdot (x+1) \cdot (x-2) \cdot (x^2+1)$$

So our final two roots are $x = \pm i$, which we can now use to write $f(x)$ as a product of 5 linear terms:

$$x^5 - 2x^4 - x + 2 = (x-1) \cdot (x+1) \cdot (x-2) \cdot (x+i) \cdot (x-i)$$