

Math 1613 - Trigonometry

Homework #10 - 2007.10.23

Due Date - 2007.11.01

Solutions

Prove the following identities.

1.

$$\sin(3\theta) = 3 \sin(\theta) - 4 \sin^3(\theta)$$

$$\begin{aligned}\sin(3\theta) &= 3 \sin(\theta) - 4 \sin^3(\theta) \\ &= 3 \sin(\theta) - 4 \sin(\theta) \sin(2\theta) \\ &= 3 \sin(\theta) - 4 \sin(\theta) \left(\frac{1 - \cos(2\theta)}{2} \right) \\ &= 3 \sin(\theta) - 2 \sin(\theta) + 2 \sin(\theta) \cos(2\theta) \\ &= \sin(\theta) + \sin(3\theta) + \sin(-\theta) \\ &= \sin(\theta) + \sin(3\theta) - \sin(\theta) \\ &= \sin(3\theta).\end{aligned}$$

2.

$$\frac{1}{2} \cot(x) - \frac{1}{2} \tan(x) = \cot(2x)$$

First we multiply everything by 2 and go from there:

$$\begin{aligned}\cot(x) - \tan(x) &= 2 \cot(2x) \\ &= 2 \frac{1 - \tan^2(x)}{2 \tan(x)} \\ &= \frac{1 - \tan^2(x)}{\tan(x)} \\ &= \frac{1}{\tan(x)} - \frac{\tan^2(x)}{\tan(x)} \\ &= \frac{1}{\tan(x)} - \tan(x) \\ &= \cot(x) - \tan(x).\end{aligned}$$

3.

$$\sin(4\theta) = 4 \sin(\theta) \cos(\theta) - 8 \sin^3(\theta) \cos(\theta)$$

Here, we use the result of problem 1.

$$\begin{aligned} \sin(4\theta) &= 4 \sin(\theta) \cos(\theta) - 8 \sin^3(\theta) \cos(\theta) \\ &= 4 \sin(\theta) \cos(\theta) - (6 \sin(\theta) - 2 \sin(3\theta) \cos(\theta)) \\ &= 4 \sin(\theta) \cos(\theta) - 6 \sin(\theta) \cos(\theta) + 2 \sin(3\theta) \cos(\theta) \\ &= 2 \sin(\theta) \cos(\theta) + 2 \sin(3\theta) \cos(\theta) \\ &= \sin(2\theta) + 2 \sin(3\theta) \cos(\theta) \\ &= \sin(2\theta) + \sin(4\theta) - \sin(2\theta) \\ &= \sin(4\theta). \end{aligned}$$

4.

$$\tan\left(\frac{B}{2}\right) = \csc(B) - \cot(B)$$

$$\begin{aligned} \tan\left(\frac{B}{2}\right) &= \csc(B) - \cot(B) \\ &= \frac{1}{\sin(B)} - \frac{\cos(B)}{\sin(B)} \\ &= \frac{1 - \cos(B)}{\sin(B)} \\ &= \tan\left(\frac{B}{2}\right). \end{aligned}$$

5.

$$\frac{\cos(5w) + \cos(w)}{\cos(w) - \cos(5w)} = \frac{\cot(2w)}{\tan(3w)}$$

$$\begin{aligned} \frac{\cos(5w) + \cos(w)}{\cos(w) - \cos(5w)} &= \frac{\cot(2w)}{\tan(3w)} \\ &= -\frac{\cos(5w) + \cos(w)}{\cos(5w) - \cos(w)} \\ &= -\frac{2 \cos(3w) \cos(2w)}{-2 \sin(3w) \sin(2w)} \\ &= \frac{2 \cos(3w) \cos(2w)}{2 \sin(3w) \sin(2w)} \\ &= \cot(3w) \cot(2w) \\ &= \frac{\cot(2w)}{\tan(3w)}. \end{aligned}$$