Math 1613 - Trigonometry

Final Exam - 2009.12.15

Name: ____________________________

Sum and Difference Identities:

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1+\tan(A)\tan(B)}$$
$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

Cofunction Identities:

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin(\theta), \quad \sin\left(\frac{\pi}{2}-\theta\right) = \cos(\theta), \quad \tan\left(\frac{\pi}{2}-\theta\right) = \cot(\theta)$$

Product-to-Sum and Sum-to-Product Identities:

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$
$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$
$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$$
$$\cos(A)\sin(B) = \frac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$$
$$\sin(A)+\sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin(A)-\sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos(A)+\cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A)-\cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Double-Angle Identities:

$$\cos(2A) = \cos^2(A)-\sin^2(A) = 1-2\sin^2(A) = 2\cos^2(A)-1$$
$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$$

Half-Angle Identities:

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}, \quad \cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{1+\cos(A)}}, \quad \tan\left(\frac{A}{2}\right) = \frac{\sin(A)}{1+\cos(A)}, \quad \tan\left(\frac{A}{2}\right) = \frac{1-\cos(A)}{\sin(A)}$$

Law of Sines:
In any triangle ABC with sides a, b and c,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}.$$

Law of Cosines:
In any triangle ABC with sides a, b and c,

$$a^2 = b^2 + c^2 - 2bc\cos(A), \quad b^2 = a^2 + c^2 - 2ac\cos(B), \quad c^2 = a^2 + b^2 - 2ab\cos(C).$$

Area of a Triangle:
In any triangle ABC with sides a, b and c, the area $\mathcal{A}$ is given by

$$\mathcal{A} = \frac{1}{2}bc\sin(A) = \frac{1}{2}ab\sin(C) = \frac{1}{2}ac\sin(B).$$

If we define the semiperimeter to be $s = \frac{1}{2}(a+b+c)$, then *Heron's formula* states

$$\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}.$$

This portion of the exam has been designated calculator free.

1. For each of the four quadrants, state the sign of the three trig function cos, sin and tan.

2. State the exact values of the six trigonometric functions for the angle $\theta = \frac{11}{6}\pi$.

3. State the exact values of the six trigonometric functions for the angle $\theta = -\frac{17}{3}\pi$.

4. Convert 145° to radians.

5. Convert the radian measure $\frac{7}{9}\pi$ to degrees.

6. Find the exact value of $\cos\left(\frac{5}{12}\pi\right)$.

7. Express $\sec(\theta) - \sin(\theta)\tan(\theta)$ as a single function of θ

8. Graph the function $y = -1 - \cos\left(2x + \frac{\pi}{3}\right)$ over a two period interval. Be sure to indicate both the period and amplitude of the function on your graph.

9. Graph the function $y = 2 - 3\cot\left(x + \frac{\pi}{4}\right)$ over a two period interval. Be sure to state the period of the function on your graph.

10. Verify the identity

$$\tan^2(x) - \sin^2(x) = (\tan(x)\sin(x))^2$$

11. Find *all* solutions to the given equation for θ on the interval $[0, 2\pi)$:

$$\sin(2\theta) = \cos(2\theta) + 1$$

12. Solve the following equation for z:

$$\cos^{-1}(z) = \sin^{-1}\left(\frac{2}{7}\right)$$

13. If $w = 2 - 4i$ and $z = 5 + i$, find $w + z$, wz and $\frac{w}{z}$.

14. Simplify i^{-341}

15. Convert the complex number, expressed in polar form as $2\text{cis}(15^\circ)$, to rectangular form.

16. Using the *Law of Sines*, show that

$$\frac{\sin(a) - \sin(b)}{\sin(a) + \sin(b)} = \frac{a - b}{a + b}.$$

It is now safe to use your calculator on the remaining problems.

17. Find B if $A = 129.7°$, $a = 127$ ft, $b = 69.8$ ft.

18. Solve for all the missing sides and angles of triangle ABC if $A = 61.6°$, $a = 78.9$ cm, $b = 86.4$ cm.

19. Given $a = 10$ and $B = 30°$, determine the values of b for which A has a) exactly one value, b) two possible values, and c) no value.