

Math 2143 - Brief Calculus with Applications

Final Exam - 2008.04.24

Due Date - 2008.05.08 - 10:00 A.M.

Name: _____

Instructions

Write each problem on its own page and be sure to show all your work. Compose your answers in a very concise and neat manner.

1. Rewrite the following function as a piecewise function that does not include absolute values.

$$f(x) = \left| \frac{6x^2 - x - 1}{x - 1} \right|$$

2. Compute the following limits.

a)

$$\lim_{x \rightarrow 0} \frac{x^4 + 5x^2}{2 - \sqrt{x^2 + 4}}$$

b)

$$\lim_{x \rightarrow \infty} \frac{4x^3 - 7x^2 + x^{\frac{3}{2}}}{12x^4 + 5x^4 - 3x^2 + 1}$$

c)

$$\lim_{x \rightarrow -\infty} \frac{-4x^5 + 12x^6 + 3x - 1 + 2x^3}{2 - 6x^4 + 17x^6 + 6x^3 + 12x}$$

d)

$$\lim_{x \rightarrow 1^-} \left| \frac{x^2 + x - 2}{x - 3} \right|$$

e)

$$\lim_{x \rightarrow 1^+} \left| \frac{x^2 + x - 2}{x - 3} \right|$$

3. Compute the tangent line to $f(x) = e^{2x}(x^2 + 3x - 1)$ at $x = 0$.

4. Compute the following derivatives.

a)

$$\frac{d}{dx} e^{2e^{4x}}$$

b)

$$\frac{d}{dz} \ln(e^{z^2} + \ln(2z)) \cdot z^2$$

c)

$$\frac{d}{dw} \frac{2w^2 + 4w \ln(w + 1)}{3w^3 e^{4w} - 1}$$

d)

$$\frac{d}{dy} \sqrt{(1-3y)^{\frac{2}{3}}(1+3y)^{\frac{1}{3}}}$$

e)

$$\frac{d}{dt} \ln(t^2 + 1) e^{2t-7} \left(t^2 + 3t - \frac{1}{t^3} + \sqrt{t-1} \right)$$

5. Sketch the graphs of the following three functions. For each, be sure to include the following information in your work:

- A) Domain
- B) x - and y -intercepts
- C) Asymptotes (horizontal, vertical and slant)
- D) Critical points
- E) Intervals of increase and decrease
- F) Inflection points
- G) Intervals of concavity

a)

$$f(x) = \frac{x^2}{\sqrt{x+1}}$$

b)

$$g(x) = \ln(4 - x^2)$$

c)

$$h(x) = \frac{x^3}{x^2 + 1}$$

6. Find the area of the largest rectangle that can be inscribed inside a circle of radius R .

7. A store has been selling 200 DVD players per week at \$350. A market survey indicates that for each \$10 of rebate offered to buyers, the number of players sold will increase by 20 per week.

- a) Find the demand function.
- b) Find the revenue function.
- c) How large of a rebate should the store offer to maximize its revenue?

8. A can of pop at room temperature (72° F) is placed in a fridge where the temperature is 44° F. After half an hour the pop has cooled to 61° F.

- a) What is the temperature of the can of pop after another half hour?
- b) How long does it take for the pop to cool to 50° F?

9. Compute the following integrals.

a)

$$\int \frac{17t^8}{7t^9 + 3} dt$$

b)

$$\int_{-1}^1 \frac{3}{x+7} dx$$

c)

$$\int_{-2}^1 t^5 (6t^6 - 4)^7 dt$$

d)

$$\int \ln \left(\frac{x+1}{x+8} \right) dx$$

e)

$$\int \frac{t^4 \ln(t^5 + 3)}{t^5 + 3} dt$$

10. Find the average value of $h(t) = 7t^2 + 3t - 1$ for $-1 \leq t \leq 1$.

11. Rewrite the following statement and sign your name to it:

“I hereby swear that all the work that appears on this written exam is completely my own, and I have not discussed any portion of this exam with any one.”