Math 2143 - Brief Calculus with Applications

Homework #4 - 2008.02.07

Due Date - 2008.02.14

Solutions

1. Find that value of a such that $f(x)$ defined below is continuous on $\mathbb{R}$.

$$f(x) = \begin{cases} x+3, & x < -1 \\ x^2 + ax - 2, & x \geq -1 \end{cases}$$

First, we compute the limit from the left:

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} x + 3 = -1 + 3 = 2$$

Next, we compute the limit from the right:

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} x^2 + ax - 2 = -1 - a$$

Setting these equal gives

$$2 = -1 - a \longrightarrow a = -3.$$

2. Compute the following limits:

a) $\lim_{x \to -5} \frac{x^2-3x-10}{x-5}$

$$\lim_{x \to -5} \frac{x^2 - 3x - 10}{x - 5} = \lim_{x \to -5} \frac{(-5)^2 - 3(-5) - 10}{-5 - 5} = -3$$

b) $\lim_{x \to 5} \frac{x^2-3x-10}{x-5}$

$$\lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5} = \lim_{x \to 5} \frac{(x-5)(x+2)}{x-5} = \lim_{x \to 5} \frac{x-5}{x-5} \cdot (x+2) = \lim_{x \to 5} (x+2) = 7$$

c) $\lim_{x \to 5^+} \frac{x^2-3x-10}{|x-5|}$

The first thing we need to do is figure out how to get rid of the absolute values. Notice that

$$|x - 5| = \begin{cases} x - 5, & x > 5 \\ -(x-5), & x \leq 5, \end{cases}$$

so

$$\lim_{x \to 5^+} \frac{x^2 - 3x - 10}{|x - 5|} = \lim_{x \to 5^+} \frac{x^2 - 3x - 10}{x - 5} = 7$$

since we calculated this in part b).

d) $\lim_{x \to 5^-} \frac{x^2-3x-10}{|x-5|}$

Here, we can use the result from parts b) and c) and pull out the negative sign to get:

$$\lim_{x \to 5^-} \frac{x^2 - 3x - 10}{|x - 5|} = \lim_{x \to 5^+} \frac{x^2 - 3x - 10}{-(x - 5)} = -7$$

e) $\lim_{x \to 5} \frac{x^2-3x-10}{|x-5|}$

From parts c) and d), we get

$$\lim_{x \to 5} \frac{x^2 - 3x - 10}{|x - 5|} \text{ Does Not Exist.}$$

3. Determine the intervals over which the following functions are continuous.

a) $f(x) = \begin{cases} \frac{x^2-3x-4}{x-4}, & x \neq 4 \\ 2, & x = 4 \end{cases}$

The function is clearly continuous for $x \neq 4$. So now we need to figure out what happens as $x = 4$. So

$$\lim_{x\to 4} f(x) = \lim_{x\to 4} \frac{x^2 - 3x - 4}{x - 4} = \lim_{x\to 4} \frac{(x+1)(x-4)}{x-4} = \lim_{x\to 4}(x+1) = 5.$$

Since

$$\lim_{x\to 4} f(x) = 5 \neq 2,$$

the function is not continuous at $x = 4$.

b) $g(x) = \begin{cases} \frac{x^2-3x}{x}, & x \neq 0 \\ -3, & x = 0 \end{cases}$

The function is continuous for $x \neq 0$, so now we focus on $x = 0$.

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{x^2 - 3x}{x} = \lim_{x\to 0} \frac{x}{x}(x-3) = \lim_{x\to 0}(x-3) = -3$$

Since

$$\lim_{x\to 0} g(x) = -3 = g(0),$$

the function is continuous at $x = 0$.