Math 2143 - Brief Calculus with Applications

Homework #5 - 2008.02.12

Due Date - 2008.02.19

Solutions

Compute the following limits. SHOW ALL YOUR WORK.

1.

$$\begin{aligned}\lim_{x\to 3}\frac{x^2-4x+3}{x^2+x-12} &= \lim_{x\to 3}\frac{(x-3)(x-1)}{(x+4)(x-3)}\\ &= \lim_{x\to 3}\frac{x-3}{x-3}\cdot\lim_{x\to 3}\frac{x-1}{x+4}\\ &= 1\cdot\lim_{x\to 3}\frac{x-1}{x+4}\\ &= \frac{2}{7}\end{aligned}$$

2.

$$\begin{aligned}\lim_{x\to 1}\frac{x^4-1}{x-1} &= \lim_{x\to 1}\frac{(x-1)(x^3+x^2+x+1)}{x-1}\\ &= \lim_{x\to 1}\frac{x-1}{x-1}\cdot\lim_{x\to 1}(x^3+x^2+x+1)\\ &= 1\cdot\lim_{x\to 1}(x^3+x^2+x+1)\\ &= 4\end{aligned}$$

3.

$$\begin{aligned}\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} &= \lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}\\ &= \lim_{x\to 4}\frac{x-4}{x-4}\cdot\frac{1}{\sqrt{x}+2}\\ &= \lim_{x\to 4}\frac{x-4}{x-4}\cdot\lim_{x\to 4}\frac{1}{\sqrt{x}+2}\\ &= 1\cdot\lim_{x\to 4}\frac{1}{\sqrt{x}+2}\\ &= \frac{1}{4}\end{aligned}$$

4.

$$\begin{aligned}\lim_{x\to 2}\frac{x-2}{x^3-4x} &= \lim_{x\to 2}\frac{x-2}{x(x-2)(x+2)}\\ &= \lim_{x\to 2}\frac{x-2}{x-2}\cdot\lim_{x\to 2}\frac{1}{x(x+2)}\\ &= 1\cdot\lim_{x\to 2}\frac{1}{x(x+2)}\\ &= \frac{1}{8}\end{aligned}$$

5.

$$\begin{aligned}
\lim_{h\to 0} \frac{\frac{1}{(h+2)^2} - \frac{1}{4}}{h} &= \lim_{h\to 0} \frac{\frac{4-(h+2)^2}{4(h+2)^2}}{h} \\
&= \lim_{h\to 0} \frac{4-(h+2)^2}{4h(h+2)^2} \\
&= \lim_{h\to 0} \frac{4-h^2-4h-4}{4h(h+2)^2} \\
&= \lim_{h\to 0} \frac{h}{h} \cdot \lim_{h\to 0} \frac{-h-4}{4(h+2)^2} \\
&= 1 \cdot \lim_{h\to 0} \frac{-h-4}{4(h+2)^2} \\
&= -\frac{1}{4}
\end{aligned}$$

6.

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} &= \lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \\
&= \lim_{x\to 0} \frac{(1+x)-(1-x)}{x(\sqrt{1+x}+\sqrt{1-x})} \\
&= \lim_{x\to 0} \frac{x}{x} \cdot \lim_{x\to 0} \frac{2}{\sqrt{1+x}+\sqrt{1-x}} \\
&= 1 \cdot \lim_{x\to 0} \frac{2}{\sqrt{1+x}+\sqrt{1-x}} \\
&= 1
\end{aligned}$$

7.

$$\begin{aligned}
\lim_{h\to 0} \frac{(1+h)^3-1}{h} &= \lim_{h\to 0} \frac{1+3h+3h^2+h^3-1}{h} \\
&= \lim_{h\to 0} \frac{h}{h} \cdot \lim_{h\to 0} (3+3h+h^2) \\
&= 1 \cdot \lim_{h\to 0} (3+3h+h^2) \\
&= 3
\end{aligned}$$

8.

$$\begin{aligned}
\lim_{x\to 27} \frac{x-27}{x^{\frac{1}{3}}-3} &= \lim_{x\to 27} \frac{(x^{\frac{1}{3}}-3)(x^{\frac{2}{3}}+3x^{\frac{1}{3}}+9)}{x^{\frac{1}{3}}-3} \\
&= \lim_{x\to 27} \frac{x^{\frac{1}{3}}-3}{x^{\frac{1}{3}}-3} \cdot \lim_{x\to 27} (x^{\frac{2}{3}}+3x^{\frac{1}{3}}+9) \\
&= 1 \cdot \lim_{x\to 27} (x^{\frac{2}{3}}+3x^{\frac{1}{3}}+9) \\
&= 27
\end{aligned}$$

9.

$$\begin{aligned}
\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} &= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a} \cdot \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}} \\
&= \lim_{x\to a} \frac{x-a}{x-a} \cdot \lim_{x\to a} \frac{1}{\sqrt{x}+\sqrt{a}} \\
&= 1 \cdot \lim_{x\to a} \frac{1}{\sqrt{x}+\sqrt{a}} \\
&= \frac{1}{2\sqrt{a}}
\end{aligned}$$

10.

$$
\begin{aligned}
\lim_{h\to 0} \frac{\sqrt{a+2h}-\sqrt{a}}{h} &= \lim_{h\to 0} \frac{\sqrt{a+2h}-\sqrt{a}}{h} \cdot \frac{\sqrt{a+2h}+\sqrt{a}}{\sqrt{a+2h}+\sqrt{a}} \\
&= \lim_{h\to 0} \frac{a+2h-a}{h(\sqrt{a+2h}+\sqrt{a})} \\
&= \lim_{h\to 0} \frac{h}{h} \cdot \lim_{h\to 0} \frac{2}{\sqrt{a+2h}+\sqrt{a}} \\
&= 1 \cdot \lim_{h\to 0} \frac{2}{\sqrt{a+2h}+\sqrt{a}} \\
&= \frac{1}{\sqrt{a}}
\end{aligned}
$$