Math 2143 - Brief Calculus with Applications

Homework #6 - 2008.02.21

Due Date - 2008.02.28

Solutions

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1. Factor out at least one term from each of the following expressions.

a) $x^6 - 2^6$

$$x^6 - 2^6 = (x-2)(x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32)$$

b) $3^8 - x^4$

$$\begin{aligned} 3^8 - x^4 &= 9^4 - x^4 \\ &= (9-x)(729 + 81x + 9x^2 + x^3) \end{aligned}$$

c) $a^7 - 128c^7$

$$\begin{aligned} a^7 - 128c^7 &= a^7 - (2c)^7 \\ &= (a-2c)(a^6 + 2ca^5 + 4c^2a^4 + 8c^3a^3 + 16c^4a^2 + 32c^5a + 64c^6) \end{aligned}$$

d) $\frac{1}{t^3} - x^3$

$$\begin{aligned} \frac{1}{t^3} - x^3 &= \left(\frac{1}{t}\right)^3 - x^3 \\ &= \left(\frac{1}{t} - x\right)\left(\frac{1}{t^2} + \frac{x}{t} + x^2\right) \end{aligned}$$

2. Expand the following expressions completely.

a) $(a+b)^8$

$$(a+b)^8 = a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + a^8$$

b) $\left(3 + \frac{1}{x}\right)^5$

$$\left(3 + \frac{1}{x}\right)^5 = 3^5 + 5\frac{3^4}{x} + 10\frac{3^3}{x^2} + 10\frac{3^2}{x^3} + 5\frac{3}{x^4} + \frac{1}{x^5}$$

c) $\left(3 - \frac{1}{x}\right)^5$

$$\begin{aligned} \left(3 - \frac{1}{x}\right)^5 &= \left(3 + \left(-\frac{1}{x})\right)\right)^5 \\ &= 3^5 - 5\frac{3^4}{x} + 10\frac{3^3}{x^2} - 10\frac{3^2}{x^3} + 5\frac{3}{x^4} - \frac{1}{x^5} \end{aligned}$$

d) $\left(\frac{2}{a} + \frac{b}{3}\right)^4$

$$\left(\frac{2}{a} + \frac{b}{3}\right)^4 = \left(\frac{2}{a}\right)^4 + 4\left(\frac{2}{a}\right)^3\left(\frac{b}{3}\right) + 6\left(\frac{2}{a}\right)^2\left(\frac{b}{3}\right)^2 + 4\left(\frac{2}{a}\right)\left(\frac{b}{3}\right)^3 + \left(\frac{b}{3}\right)^4$$

3. Find an equation of the tangent line to the graph of $f(x) = x^3 + 2x - \frac{3}{\sqrt{x}}$ at $x = 1$.

First, note that $f(1) = 0$, so our point is $(1, 0)$. The slope is $f'(1)$, so we compute $f'(x)$ first.

$$f'(x) = 3x^2 + 2 + \frac{3}{2}x^{-\frac{3}{2}}$$

and $f'(1) = \frac{13}{2}$, so our equation is

$$y - 0 = \frac{13}{2}(x - 1).$$